Technische Universität München **Winter term 2017/18**
I7
Prof. J. Esparza / Dr. M. Blondin **06.01.2018**

Automata and Formal Languages — Homework 11

Due 16.01.2018

Exercise 11.1
Let $\inf(w)$ denote the set of letters occurring infinitely often in the infinite word w. Give Büchi automata and ω-regular expressions for the following ω-languages over $\Sigma = \{a, b, c\}$:

(a) $L_1 = \{w \in \Sigma^\omega : \inf(w) \subseteq \{a, b\}\}$,

(b) $L_2 = \{w \in \Sigma^\omega : \inf(w) = \{a, b\}$,

(c) $L_3 = \{w \in \Sigma^\omega : \{a, b\} \subseteq \inf(w)\}$,

(d) $L_4 = \{w \in \Sigma^\omega : \inf(w) = \{a, b, c\}\}$.

Exercise 11.2
Give *deterministic* Büchi automata recognizing the following ω-languages over $\Sigma = \{a, b, c\}$:

(a) $L_1 = \{w \in \Sigma^\omega : w$ contains at least one $c\}$,

(b) $L_2 = \{w \in \Sigma^\omega :$ in w, every a is immediately followed by a $b\}$,

(c) $L_3 = \{w \in \Sigma^\omega :$ in w, between two successive a's there are at least two b's$\}$.

Exercise 11.3
Give *deterministic* Rabin automata, Muller automata and parity automata for the following language:

$$L = \{w \in \{a, b\}^\omega : w \text{ contains finitely many } a\text{'s}\}.$$

Exercise 11.4
Prove or disprove:

(a) For every Büchi automaton A, there exists a Büchi automaton B with a single initial state and such that $L_\omega(A) = L_\omega(B)$;

(b) For every Büchi automaton A, there exists a Büchi automaton B with a single accepting state and such that $L_\omega(A) = L_\omega(B)$;

(c) There exists a Büchi automaton recognizing the finite ω-language $\{w\}$ such that $w \in \{0, 1, \ldots, 9\}^\omega$ and w_i is the i^{th} decimal of π.