Technische Universität München **Winter term 2017/18**
I7
Prof. J. Esparza / Dr. M. Blondin **01.12.2017**

Automata and Formal Languages — Homework 7

Due 05.12.2017

Exercise 7.1

Let val : $\{0,1\}^* \to \mathbb{N}$ be the function that associates to every word $w \in \{0,1\}^*$ the number val(w) represented by w in the *least significant bit first* encoding.

(a) Give a transducer that doubles numbers, i.e. a transducer accepting

$$L_1 = \{[x, y] \in (\{0,1\} \times \{0,1\})^* : \mathrm{val}(y) = 2 \cdot \mathrm{val}(x)\}.$$

(b) Give an algorithm that takes $k \in \mathbb{N}$ as input, and that produces a transducer A_k accepting

$$L_k = \left\{[x, y] \in (\{0,1\} \times \{0,1\})^* : \mathrm{val}(y) = 2^k \cdot \mathrm{val}(x)\right\}.$$

[Hint:]

(c) Give a transducer for the addition of two numbers, i.e. a transducer accepting

$$\{[x, y, z] \in (\{0,1\} \times \{0,1\} \times \{0,1\})^* : \mathrm{val}(z) = \mathrm{val}(x) + \mathrm{val}(y)\}.$$

(d) For every $k \in \mathbb{N}_{>0}$, let

$$X_k = \{[x, y] \in (\{0,1\} \times \{0,1\})^* : \mathrm{val}(y) = k \cdot \mathrm{val}(x)\}.$$

Sketch an algorithm that takes as input transducers A and B, accepting respectively X_a and X_b for some $a, b \in \mathbb{N}_{>0}$, and that produces a transducer C accepting X_{a+b}. [Hint:]

(e) Let $k \in \mathbb{N}_{>0}$. Using (b) and (d), how can you build a transducer accepting X_k?

(f) Show that the following language has infinitely many residuals, and hence that it is not regular:

$$\left\{[x, y] \in (\{0,1\} \times \{0,1\})^* : \mathrm{val}(y) = \mathrm{val}(x)^2\right\}.$$

Exercise 7.2

Consider transducers whose transitions are labeled by elements of $(\Sigma \cup \{\varepsilon\}) \times (\Sigma^* \cup \{\varepsilon\})$. Intuitively, each transition reads one or zero letter and writes a word of arbitrary length. Such a transducer can be used to perform operations on strings, e.g. upon reading `singing in the rain` it could write `Singing In The Rain`.

Sketch such ε-transducers for the following operations, each of which is informally defined by means of three examples. For each example, when the transducer reads the string on the left, it should write the string on the right. You may assume that the alphabet Σ consists of $\{a, b, \ldots, z, A, B, \ldots, Z\}$, a whitespace symbol, and an end-of-line symbol. Moreover, you may assume that every string ends with an end-of-line symbol and contains no other occurrence of the end-of-line symbol.

(a)

Input	Output
European Research Council	ERC
Technical University of Munich	TUM
FC Bayern	FCB

(b)

Input	Output
Finite automata rule	Finite automata rule
Transducers are fun	Transducers are fun
regular expressions are the best	regular expressions are the best

(c)

Input	Output
Hello world	HHEELLOO WWOORRLLDD
This is Sparta	TTHHIISS IISS SSPPAARRTTAA
over nine thousand	OOVVEERR NNIINNEE TTHHOOUUSSAANNDD

(d) For this exercise, Σ is extended with $\{,, \cdot\}$.

Input	Output
Ada Lovelace	Lovelace, A.
Alan Turing	Turing, A.
Donald Knuth	Knuth, D.

(e) For this exercise, Σ is extended with $\{0, 1, \ldots, 9, (,), +\}$.

Input	Output
004989273452	+49 89 273452
(00)4989273452	+49 89 273452
273452	+49 89 273452