Automata and Formal Languages — Homework 6

Due 27.11.2017

Exercise 6.1

(a) Build B_p and C_p for the word pattern $p = abrababra$.

(b) How many transitions are taken when reading $t = abrar$ in B_p and C_p respectively?

(c) Let $n > 0$. Find a text $t \in \{a, b\}^*$ and a word pattern $p \in \{a, b\}^*$ such that testing whether p occurs in t takes n transitions in B_p and $2n - 1$ transitions in C_p.

Exercise 6.2

(a) Let $n \in \mathbb{N}$ be such that $n \geq 2$. Show that $L_n = \{w \in \{a, b\}^* : |w| \equiv 0 \pmod{n}\}$ has exactly n residuals, without constructing any automaton for L_n.

(b) Consider the following "proof" showing that L_2 is non regular:

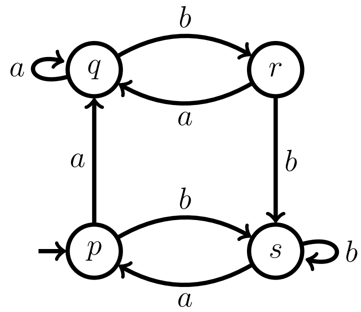
> Let $i, j \in \mathbb{N}$ be such that i is even and j is odd. By definition of L_2, we have $\varepsilon \in (L_2)^{a^i}$ and $\varepsilon \notin (L_2)^{a^j}$. Therefore, the a^i-residual and a^j-residual of L_2 are distinct. Since there are infinitely many even numbers i and odd numbers j, this implies that L_2 has infinitely many residuals, and hence that L_2 is not regular. □

Language L_2 is regular, so this "proof" must be incorrect. Explain what is wrong with the "proof".

(c) Show that $P = \{w \in \{a, b\}^* : |w| \text{ is a power of } 2\}$ is not regular, by showing that P has infinitely many residuals. Is $P \cap \{a\}^*$ also non regular?

Exercise 6.3
Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA. A word $w \in \Sigma^*$ is a *synchronizing word* of A if reading w from any state of A leads to a common state, i.e. if there exists $q \in Q$ such that for every $p \in Q$, $p \xrightarrow{w} q$. A DFA is *synchronizing* if it has a synchronizing word.

(a) Show that the following DFA is synchronizing:



(b) Give a DFA that is not synchronizing.

(c) Give an exponential time algorithm to decide whether a DFA is synchronizing. [Hint:]

(d) Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA. We say that A is (p,q)-*synchronizing* if there exist $w \in \Sigma^*$ and $r \in Q$ such that $p \xrightarrow{w} r$ and $q \xrightarrow{w} r$. Show that A is synchronizing if and only if it is (p,q)-synchronizing for every $p, q \in Q$.

(e) Give a polynomial time algorithm to test whether a DFA is synchronizing. [Hint:]

(f) Show, from (d), that every synchronizing DFA with n states has a synchronizing word of length at most $(n^2 - 1)(n - 1)$. [Hint:]

(g) Show that the upper bound obtained in (f) is not tight by finding a synchronizing word of length $(4 - 1)^2$ for the following DFA: