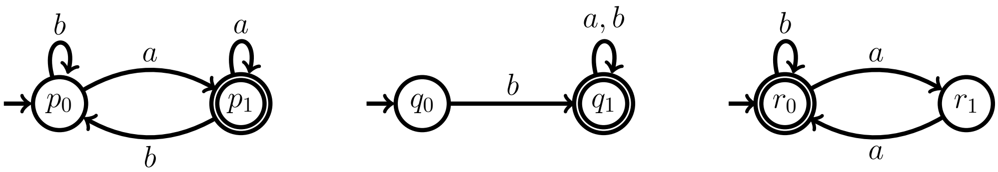
Technische Universität München **Winter term 2017/18**
I7
Prof. J. Esparza / Dr. M. Blondin **10.11.2017**

Automata and Formal Languages — Homework 4

Due 14.11.2017

Exercise 4.1
Consider the following DFAs A, B and C:



Use pairings to decide *algorithmically* whether $L(A) \cap L(B) \subseteq L(C)$.

Exercise 4.2
Let $L \subseteq \Sigma^*$ be a language accepted by an NFA A. For every $u, v \in \Sigma^*$, we say that $u \preceq v$ if and only if u can be obtained by deleting zero, one or multiple letters of v. For example, $abc \preceq abca$, $abc \preceq acbac$, $abc \preceq abc$, $\varepsilon \preceq abc$ and $aab \not\preceq acbac$. Give an NFA-ε for each of the following languages:

(a) $\downarrow L = \{w \in \Sigma^* : w \preceq w' \text{ for some } w' \in L\}$,

(b) $\uparrow L = \{w \in \Sigma^* : w' \preceq w \text{ for some } w' \in L\}$,

(c) $\sqrt{L} = \{w \in \Sigma^* : ww \in L\}$,

(d) $\text{Cyc}(L) = \{vu \in \Sigma^* : uv \in L\}$.

Exercise 4.3
Let Σ_1 and Σ_2 be alphabets. A *morphism* is a function $h : \Sigma_1^* \to \Sigma_2^*$ such that $h(\varepsilon) = \varepsilon$ and $h(uv) = h(u) \cdot h(v)$ for every $u, v \in \Sigma_1^*$. In particular, $h(a_1 a_2 \cdots a_n) = h(a_1)h(a_2) \cdots h(a_n)$ for every $a_1, a_2, \ldots, a_n \in \Sigma$. Hence, a morphism h is entirely determined by its image over letters.

(a) Let $L \subseteq \Sigma_1^*$ be accepted by some NFA A_1. Give an NFA-ε B_2 that accepts $h(L) = \{h(w) : w \in L\}$.

(b) Let $L \subseteq \Sigma_2^*$ be accepted by some NFA A_2. Give an NFA B_1 that accepts $h^{-1}(L) = \{w \in \Sigma_1^* : h(w) \in L\}$.

(c) Show that $L = \{(aab)^n e^m (cad)^n ef(fe)^n : m, n \in \mathbb{N}\}$ is not regular by using the fact that $\{a^n b^n : n \in \mathbb{N}\}$ is not regular.

Exercise 4.4

For every $n \in \mathbb{N}$, let $L_n \subseteq \{a, b\}^*$ be the language described by the regular expression $(a+b)^* a (a+b)^n b (a+b)^*$.

(a) Give an NFA A_n with $n + 3$ states that accepts L_n.

(b) Decide *algorithmically* whether $baabba \in L(A_2)$ and $baabaa \in L(A_2)$.

(c) If you make final and non final states of A_n respectively non final and final, do you obtain an NFA that accepts $\overline{L_n}$? Justify your answer.

(d) Show that $ww \notin L_n$ for every $w \in \{a, b\}^{n+1}$.

(e) Show that any NFA accepting $\overline{L_n}$ has at least 2^{n+1} states. [Hint:]