Technische Universität München **Winter term 2017/18**
I7
Prof. J. Esparza / Dr. M. Blondin **02.11.2017**

Automata and Formal Languages — Homework 3

Due 07.11.2017

Exercise 3.1
Let $M_n = \{w \in \{0,1\}^* : \text{msbf}(w) \text{ is a multiple of } n\}$ (see Exercise #1.2) and let $L_{\text{pal}} = \{w \in \Sigma^* : w = w^R\}$ where Σ is some finite alphabet.

(a) Show that M_3 has (exactly) three residuals, i.e. show that $|\{(M_3)^w : w \in \{0,1\}^*\}| = 3$.

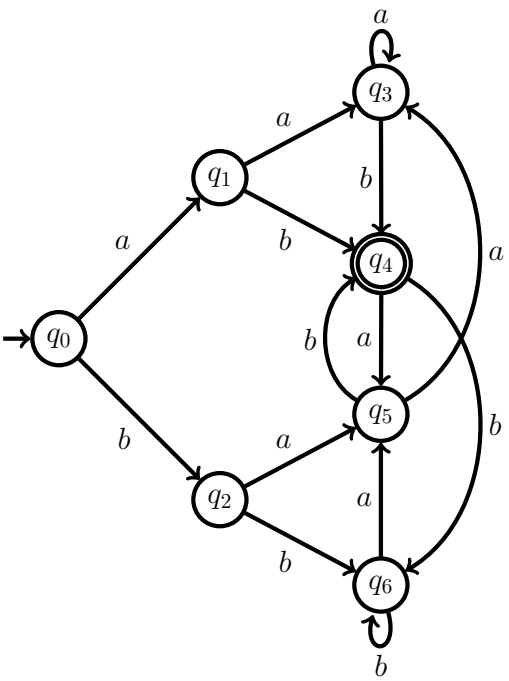
(b) Show that M_4 has less than four residuals.

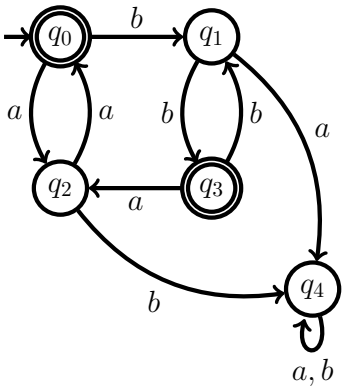
(c) ★ Show that M_p has (exactly) p residuals for every prime number p. You may use the fact that, by Fermat's little theorem, $2^{p-1} \equiv 1 \pmod{p}$. [Hint:]

(d) Show that L_{pal} has infinitely many residuals whenever $|\Sigma| \geq 2$.

(e) Show that L_{pal} is regular for $\Sigma = \{a\}$. Is L_{pal} also regular for larger alphabets?

Exercise 3.2
Let A and B be respectively the following DFAs:



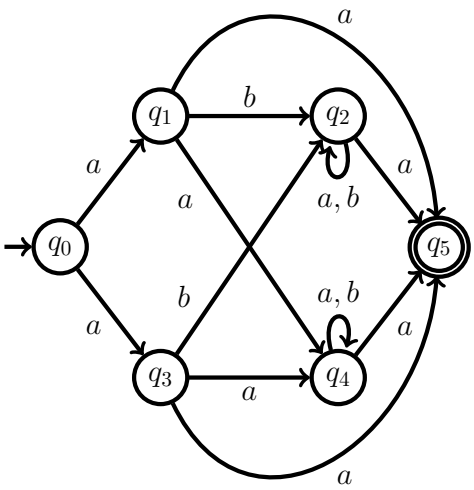


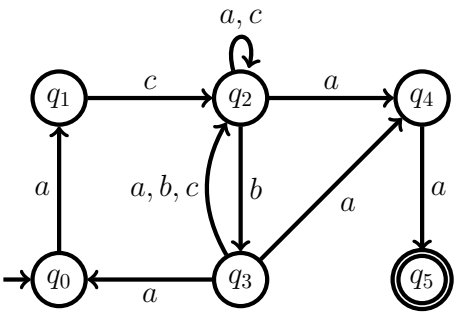
(a) Compute the language partitions of A and B.

(b) Construct the quotients of A and B with respect to their language partitions.

(c) Give regular expressions for $L(A)$ and $L(B)$.

Exercise 3.3
Let A and B be respectively the following NFAs:





(a) Compute the coarsest stable refinements (CSR) of A and B.

(b) Construct the quotients of A and B with respect to their CSRs.

(c) Show that

$$L(A) = \{w \in \{a, b\}^* : w \text{ starts and ends with } a\}$$
$$L(B) = \{w \in \{a, b\}^* : w \text{ starts with } ac \text{ and ends with } ab\}$$

(d) Are the automata obtained in (b) minimal?