Technische Universität München **Winter term 2017/18**
I7
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Automata and Formal Languages — Homework 1

Due 24.10.2017

Download `JFLAP` from www.jflap.org. We will use the *finite automata* and *regular expression* modes.

Exercise 1.1
Let $L = \{w \in \{a,b,c\}^* : w \text{ starts with } ac \text{ and ends with } ab\}$.

(a) Give an NFA that accepts L.

(b) Give a DFA that accepts L.

(c) Give a regular expression for L.

(d) Use `JFLAP` to convert your NFA of (a) and your regular expression of (c) to DFAs.

Exercise 1.2
Let $\mathrm{msbf}\colon \{0,1\}^* \to \mathbb{N}$ and $\mathrm{lsbf}\colon \{0,1\}^* \to \mathbb{N}$ be such that $\mathrm{msbf}(w)$ and $\mathrm{lsbf}(w)$ are respectively the number represented by w in the "most significant bit first" and "least significant bit first" encoding. For example,

$$\begin{aligned} \mathrm{msbf}(1010) &= 10, & \mathrm{msbf}(100) &= 4, & \mathrm{msbf}(0011) &= 3, \\ \mathrm{lsbf}(1010) &= 5, & \mathrm{lsbf}(100) &= 1, & \mathrm{lsbf}(0011) &= 12. \end{aligned}$$

For every $n \geq 2$, let us define the following languages:

$$\begin{aligned} M_n &= \{w \in \{0,1\}^* : \mathrm{msbf}(w) \text{ is a multiple of } n\}, \\ L_n &= \{w \in \{0,1\}^* : \ \mathrm{lsbf}(w) \text{ is a multiple of } n\}. \end{aligned}$$

(a) Give DFAs and regular expressions for M_2, L_2 and $M_2 \cap L_2$.

(b) Give DFAs and regular expressions for M_4, L_4 and $M_4 \cap L_4$.

(c) Give a DFA that accepts M_3. [Hint:]

(d) Give a DFA that accepts L_3. [Hint:]

(e) What can you say about $M_3 \cap L_3$?

(f) Use `JFLAP` to obtain a regular expression for M_3.

(g) Give a general DFA construction for M_n where $n \geq 2$.

Exercise 1.3
The *reverse* of a word $w \in \Sigma^*$ is defined as

$$w^R = \begin{cases} \varepsilon & \text{if } w = \varepsilon, \\ a_n a_{n-1} \cdots a_1 & \text{if } w = a_1 a_2 \cdots a_n \text{ where each } a_i \in \Sigma. \end{cases}$$

The *reverse* of a language $L \subseteq \Sigma^*$ is defined as $L^R = \{w^R : w \in L\}$.

(a) Let A be an NFA. Describe an NFA B such that $L(B) = L(A)^R$.

(b) Does your construction in (a) works for DFAs as well? More precisely, does it preserve determinism?

(c) Show that $M_n = (L_n)^R$ for every $n \geq 2$.

Exercise 1.4
Let A and B be DFAs over some alphabet Σ.

(a) Describe DFAs C and D such that $L(C) = L(A) \cup L(B)$ and $L(D) = L(A) \cap L(B)$.

(b) Prove that D is correct, i.e. that indeed $L(C) = L(A) \cap L(B)$.

(c) If A and B were NFAs, could you construct NFAs with fewer states for union and intersection? Explain your answer.