Automata and Formal Languages

Winter 2016/17

Syllabus

Course schedule

Lectures

Prof. Javier Esparza (esparza@in.tum.de)

Room:	02.13.010
Wednesday:	10:00 – 11:30
Thursday:	10:00 – 11:30

Exercises

Dr. Michael Blondin (blondin@in.tum.de)

Room:	03.09.014
Friday:	10:00 – 11:30

Course schedule

Lectures

Prof. Javier Esparza (esparza@in.tum.de)

Room:	02.13.010
Wednesday:	10:15 – 11:45?
Thursday:	10:15 – 11:45?

Exercises

Dr. Michael Blondin (blondin@in.tum.de)

Room:	03.09.014
Friday:	10:15 – 11:45?

Course content

Automata on finite words

1. Automata classes and conversions
2. Minimization and reduction
3. Boolean operations and tests
4. Operations on relations
5. Operations on finite universes: decision diagrams
6. Automata and logic
7. Pattern-matching, verification, Presburger arithmetic

Course content

Automata on infinite words

8. Automata classes and conversions
9. Boolean operations
10. Emptiness check
11. Verification using temporal logic

Grading

Written exam

- end of the term
- /40 points

Exercises not graded!

Points	Grade
[36, 40]	1,0
[34, 36)	1,3
[32, 34)	1,7
[30, 32)	2,0
[28, 30)	2,3
[26, 28)	2,7
[24, 26)	3,0
[22, 24)	3,3
[19, 22)	3,7
[17, 19)	4,0
[11, 17)	4,3
[5, 11)	4,7
[0, 5)	5,0

Material

- Lecture notes available online
- Slides available online
- No book to buy

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www7.in.tum.de > Teaching
               > Automata
               > more info
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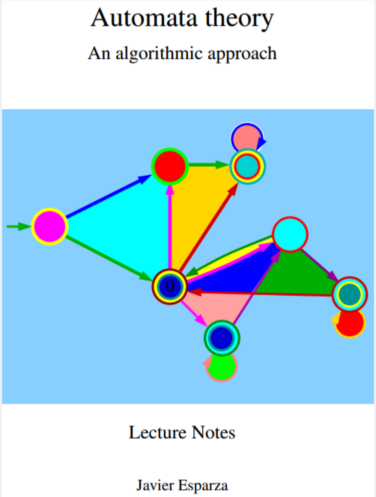

Automata theory: brief recap

Formal languages

An *alphabet* is a nonempty finite set of *letters*

e.g. $\{\mathtt{0}, \mathtt{1}\}$, $\{\mathtt{a}, \mathtt{b}, \ldots, \mathtt{z}\}$, $\{\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right], \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]\}$, $\{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$

A *(finite) word* is a finite sequence of letters

e.g. $\mathtt{1001}$, $\mathtt{hello}$, $\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]$, $\clubsuit\clubsuit\diamondsuit$, ε

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Formal languages

Let $u = a_1 \cdots a_n$ and $v = b_1 \cdots b_m$ be words

$$\begin{aligned} \textit{Concatenation:} \quad u \cdot v &= uv = a_1 \cdots a_n b_1 \cdots b_m \\ \varepsilon \cdot u &= u = u \cdot \varepsilon \end{aligned}$$

Exponentiation: $u^0 = \varepsilon$, $u^{k+1} = u^k \cdot u$

e.g. $\mathtt{a}^0 = \varepsilon$, $\mathtt{a}^1 = \mathtt{a}$, $(\mathtt{hallo})^2 = \mathtt{hallohallo}$,
$\mathtt{1}^5 = \mathtt{11111}$, $\varepsilon^{1000} = \varepsilon$, $\mathtt{ab} \cdot \mathtt{cde} = \mathtt{abcde}$

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Formal languages

Let L and L' be languages over alphabet Σ

Concatenation: $L \cdot L' \;=\; LL' \;=\; \{u \cdot v : u \in L, v \in L'\}$

Exponentiation: $L^0 = \{\varepsilon\}$, $L^{k+1} = L^k \cdot L$

Iteration: $L^* \;=\; L^0 \;\cup\; L^1 \;\cup\; L^2 \;\cup\; \cdots$

Complement: $\overline{L} = \Sigma^* \setminus L$

e.g.
$$\begin{aligned} \{\mathrm{aa}, \mathrm{bb}\} \cdot \{\mathrm{c}, \mathrm{d}\} &= \{\mathrm{aac}, \mathrm{aad}, \mathrm{bbc}, \mathrm{bbd}\}, \\ \{\mathrm{aa}, \mathrm{b}\}^* &= \{\varepsilon, \mathrm{aa}, \mathrm{b}, \mathrm{aaaa}, \mathrm{aab}, \ldots\} \end{aligned}$$

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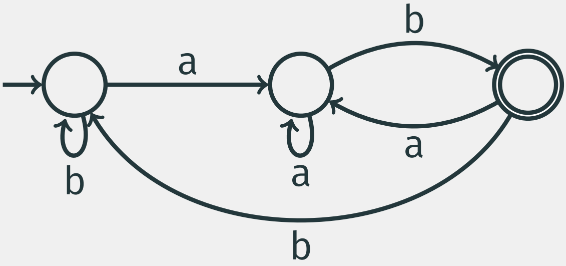
Deterministic finite automata (DFA)

- *States*: nonempty finite set Q
- *Alphabet*: nonempty finite set Σ
- *Transitions*: $\delta : Q \times \Sigma \to Q$
- *Initial state*: $q_0 \in Q$
- *Final states*: $F \subseteq Q$



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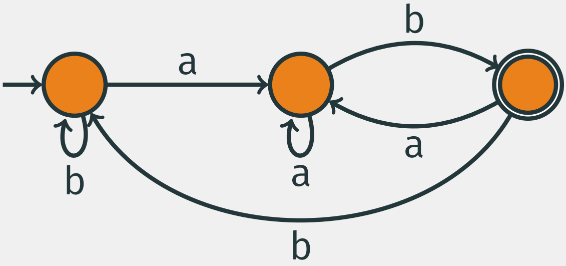

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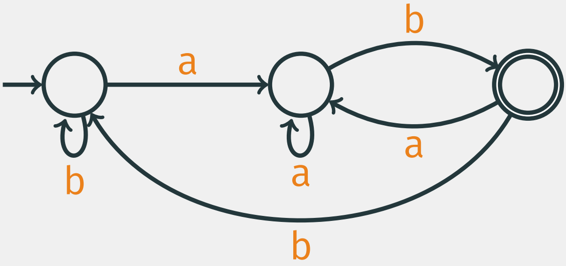

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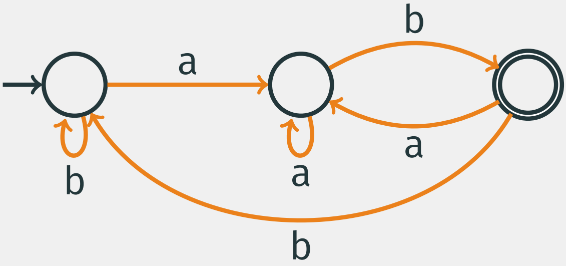

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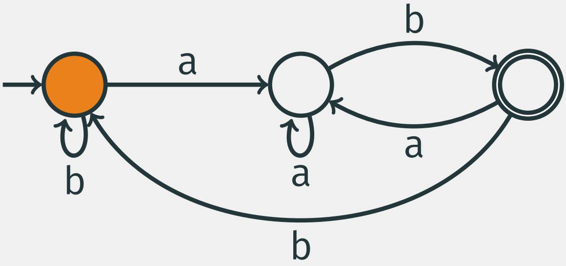

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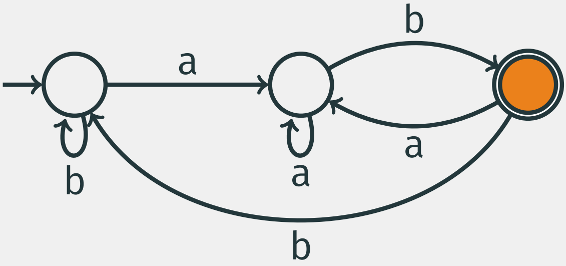
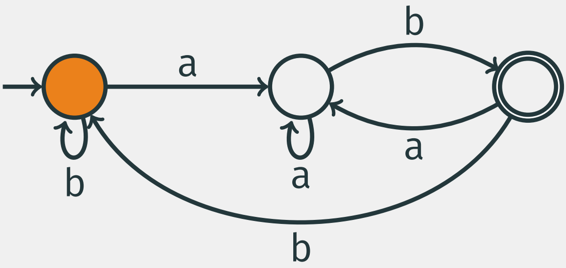


Deterministic finite automata (DFA)

w = aabab



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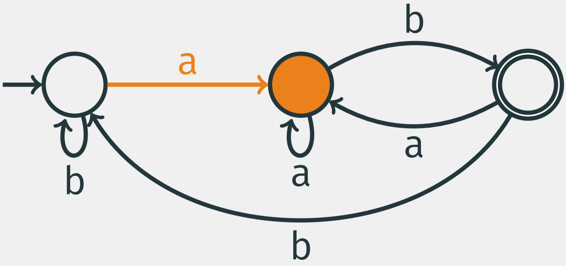

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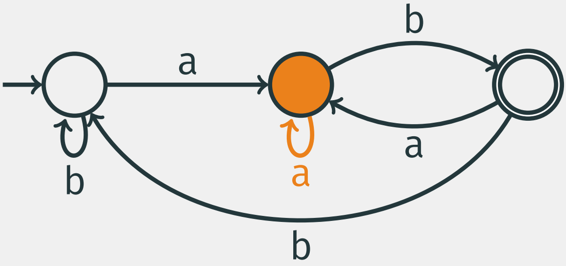

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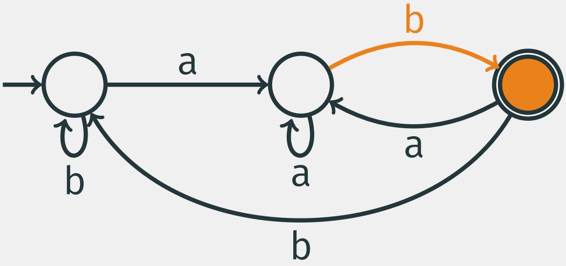

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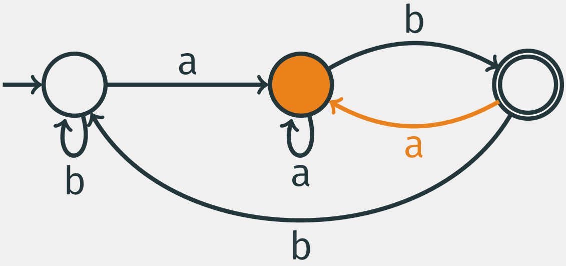
$$w = aabab$$



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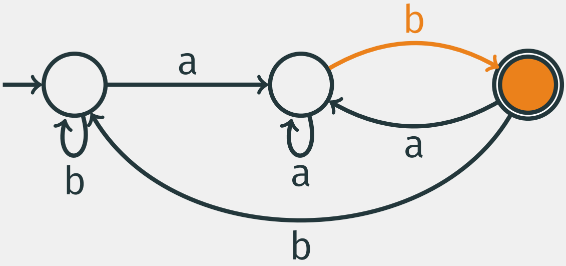
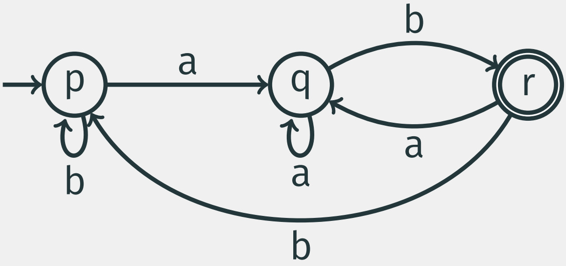


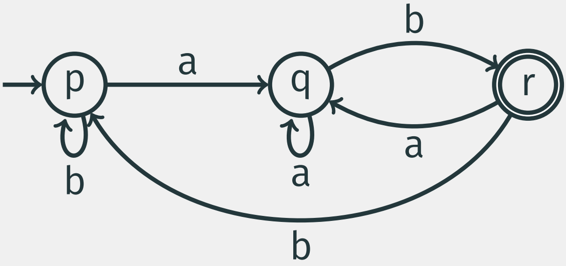
Deterministic finite automata (DFA)

$$p \xrightarrow{a} q \xrightarrow{a} q \xrightarrow{b} r \xrightarrow{a} q \xrightarrow{b} r$$



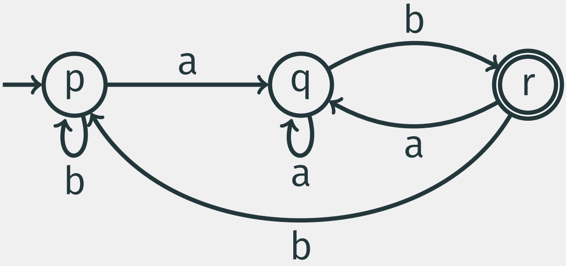
Deterministic finite automata (DFA)

$$p \xrightarrow{\text{aabab}} r$$

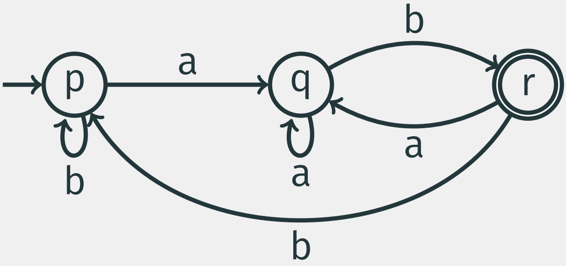


Deterministic finite automata (DFA)

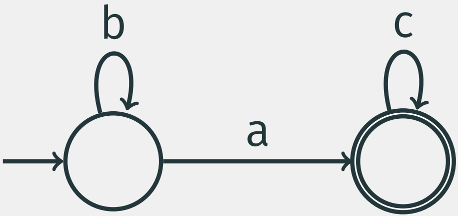
$$L(A) = \{w \in \Sigma^* : \exists q \in F \text{ s.t. } q_0 \xrightarrow{w} q\}$$



$$L(A) = \{w \in \Sigma^* : \quad w \text{ ends with ab} \quad \}$$

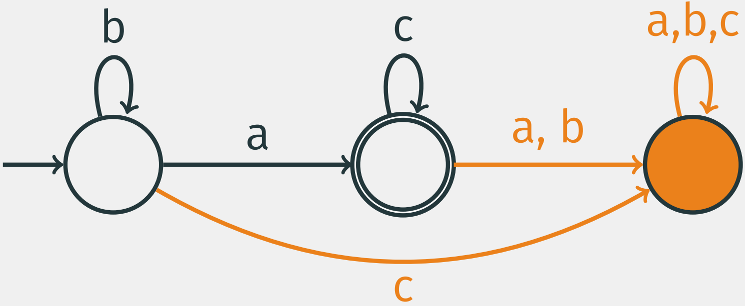


Transition function δ defined *on every* input

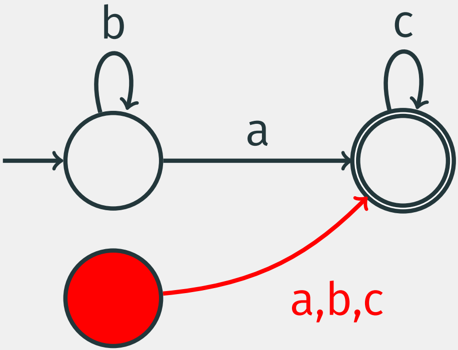


DFA: trap states and unreachable states

Transition function δ defined *on every* input



Every state *reachable* from initial state



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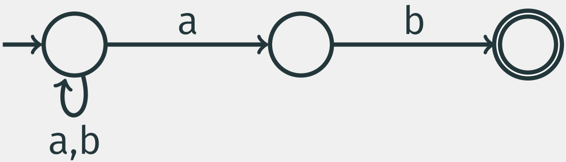

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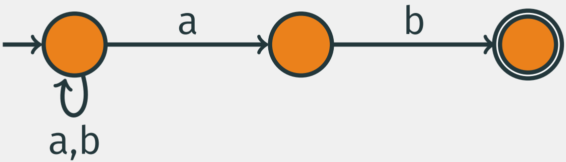

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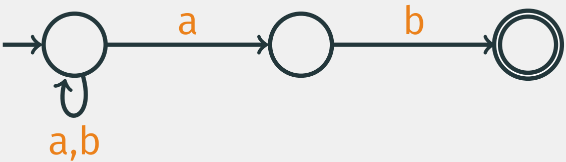

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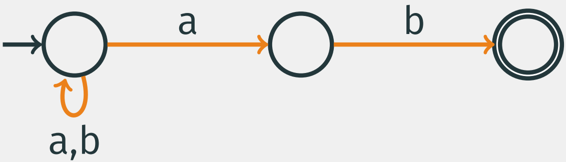

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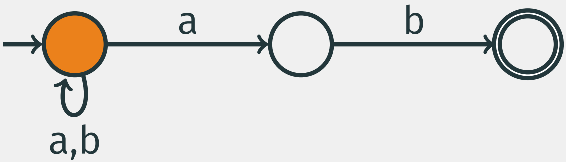

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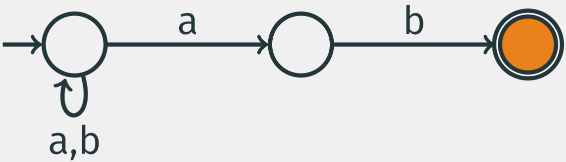

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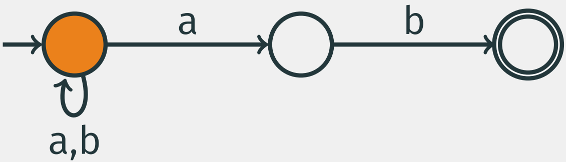
w = aab



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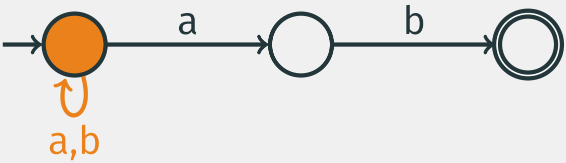

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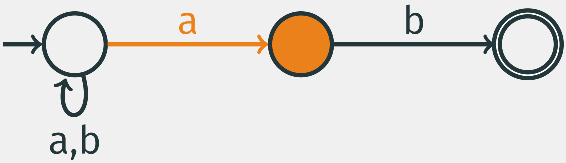

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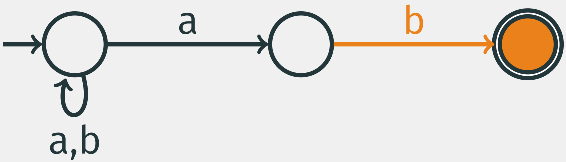

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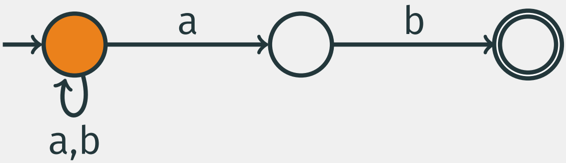

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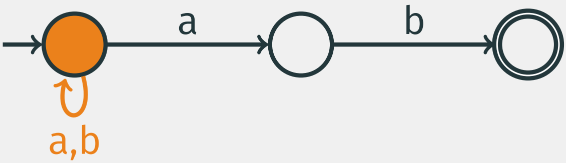

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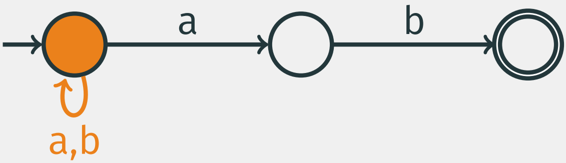

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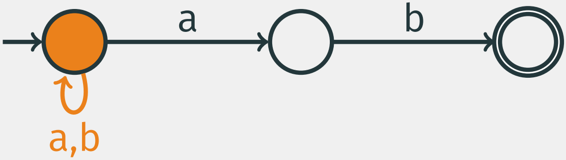
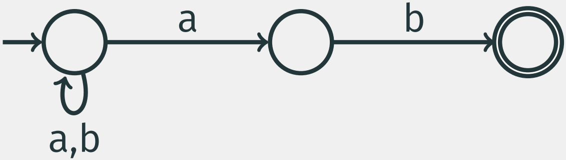


$$p_0 \xrightarrow{a_1} p_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} p_n$$

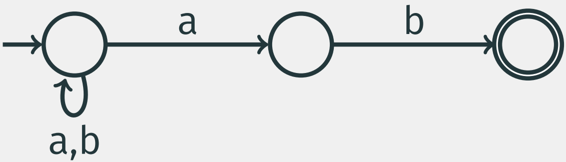
$$\Updownarrow$$

$$p_i \in \delta(p_{i-1}, a_i) \text{ for every } 0 < i \leq n$$



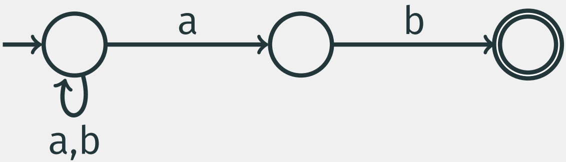
Nondeterministic finite automata (NFA)

$$L(A) = \{w \in \Sigma^* : \exists q_0 \in Q_0, q \in F \text{ s.t. } q_0 \xrightarrow{w} q\}$$



Nondeterministic finite automata (NFA)

$$L(A) = \{w \in \Sigma^* : \quad w \text{ ends with ab} \quad \}$$



Regular expressions

$$r ::= \emptyset \mid \varepsilon \mid a \mid r_1 r_2 \mid r_1 + r_2 \mid r^*$$

Regular expressions

$$r \;::=\; \emptyset \mid \varepsilon \mid a \mid r_1\, r_2 \mid r_1 + r_2 \mid r^*$$

$$
\begin{aligned}
L(\emptyset) &= \emptyset & \qquad L(r_1\, r_2) &= L(r_1) \cdot L(r_2) \\
L(\varepsilon) &= \{\varepsilon\} & \qquad L(r_1 + r_2) &= L(r_1) \cup L(r_2) \\
L(a) &= \{a\} & \qquad L(r^*) &= L(r)^*
\end{aligned}
$$

Regular expressions

$$L(\,(a+b)^*ab\,) = \{w \in \{a,b\}^* : w \text{ ends with } \texttt{ab}\}$$

$$\begin{aligned}
L(\emptyset) &= \emptyset & \quad L(r_1 r_2) &= L(r_1) \cdot L(r_2) \\
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More examples

$$L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* : w \text{ contains } \mathtt{aaa}\}$$

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Regular expression?

More examples

$$L = \{w \in \{\mathrm{a}, \mathrm{b}\}^* : w \text{ contains aaa}\}$$

Regular expression?

$$(\mathrm{a} + \mathrm{b})^*\mathrm{aaa}(\mathrm{a} + \mathrm{b})^*$$

More examples

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NFA?

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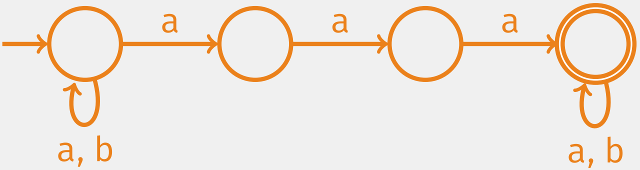

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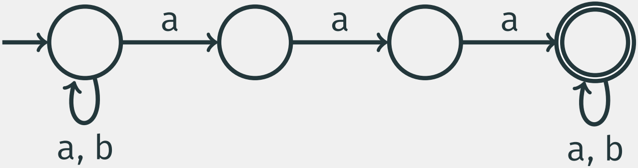
DFA?



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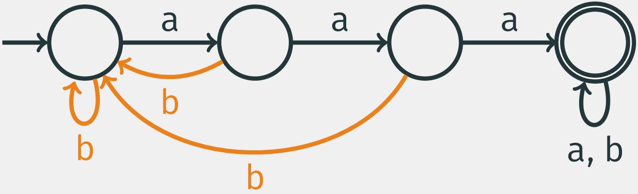

More examples

$$L = \{w \in \{0, 1\}^* : w \text{ contains an even number of } 0 \text{ or an odd number of } 1\}$$

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Regular expression?

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Regular expression?

$$(1^*01^*0)^*1^* + (0^*10^*1^*)^*0^*1^*0^*$$

More examples

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NFA?

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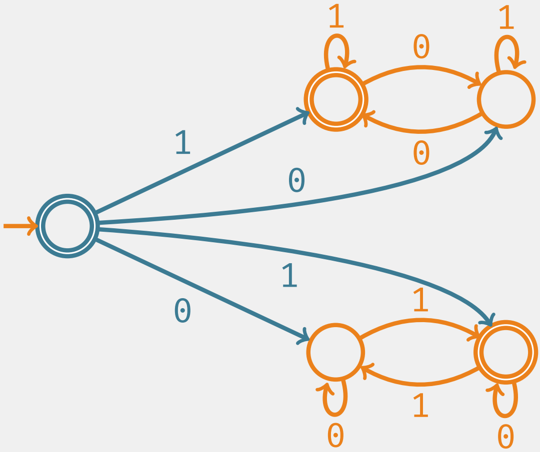
NFA?



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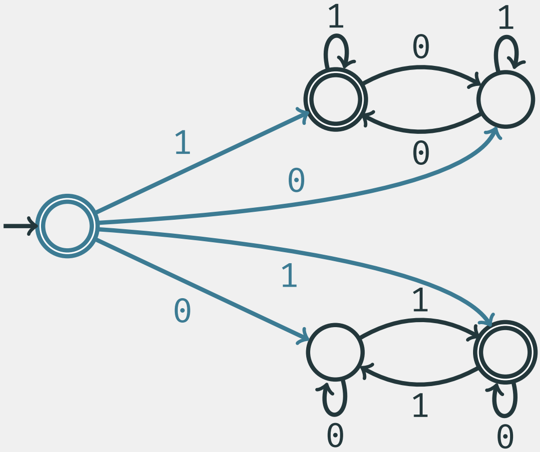
DFA?



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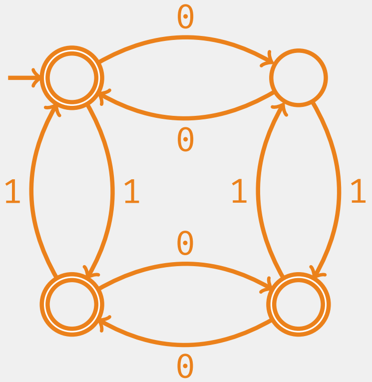
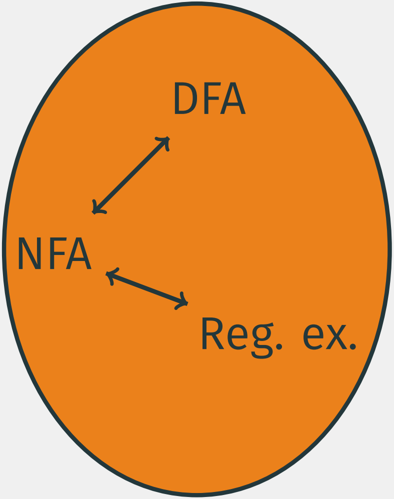
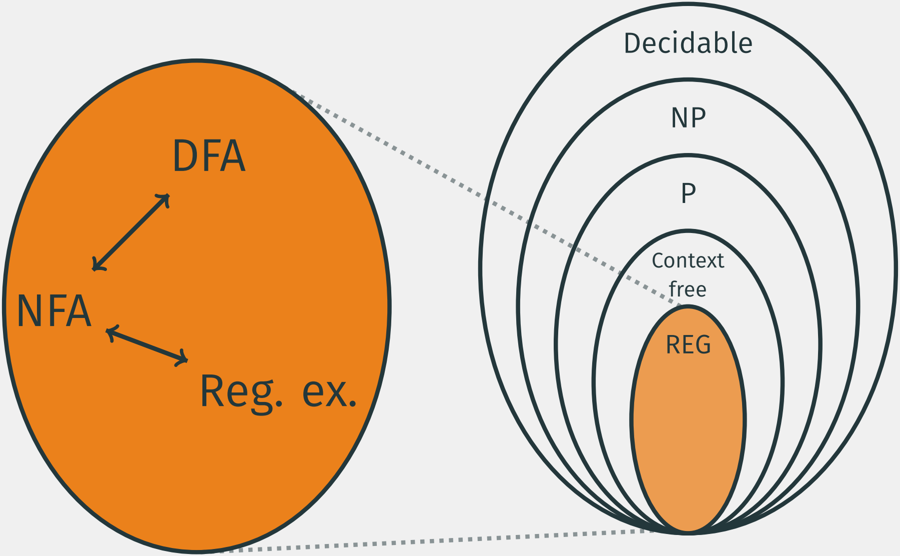
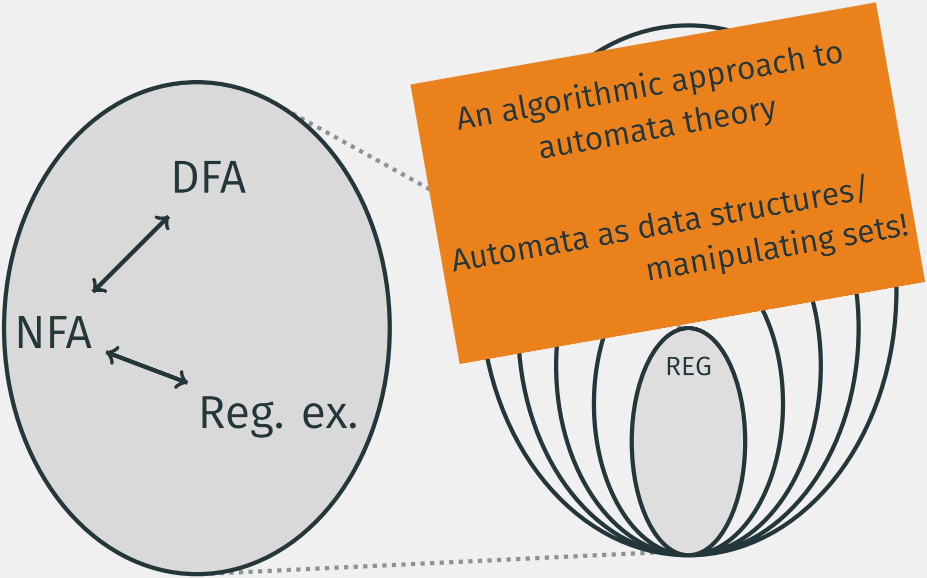



DFA
NFA
Reg. ex.

Regular languages



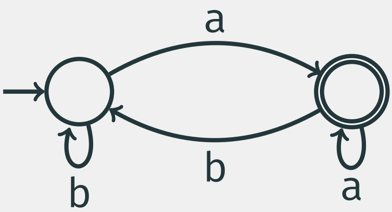
Regular languages



Beyond finite words

An *infinite word* is an infinite sequence $a_0 a_1 a_2 \cdots$ over some Σ

A *Büchi automaton* is "as an NFA", but accepts infinite words



Büchi automata

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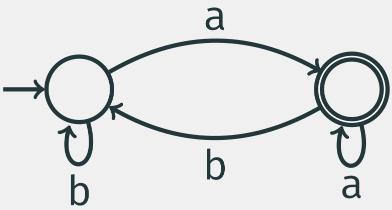

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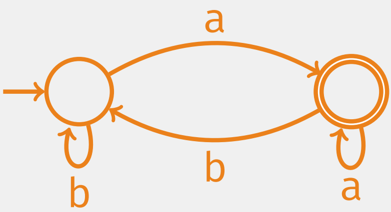

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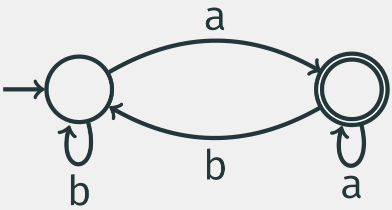


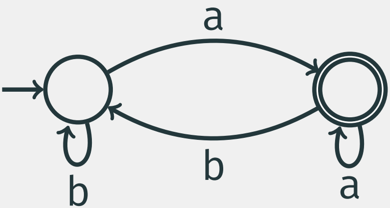
$L_\omega(A) = \{w \in \{\mathrm{a}, \mathrm{b}\}^\omega : w \text{ contains infinitely many a}\}$

Büchi automata

An *in* … over some Σ

Coming later this semester!

A *Bü* … ite words



$$L_\omega(A) = \{w \in \{\mathrm{a}, \mathrm{b}\}^\omega : w \text{ contains infinitely many a}\}$$