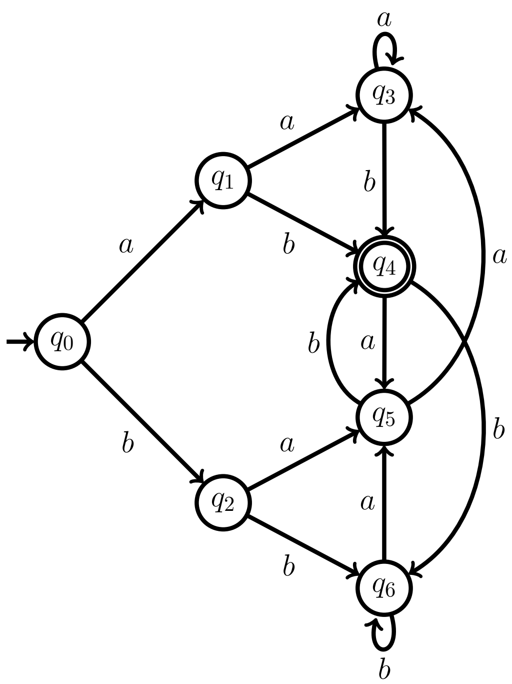
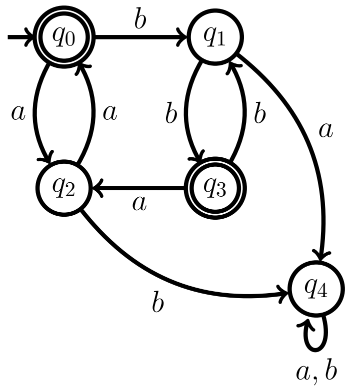
Automata and Formal Languages — Homework 3

Due 04.11.2016

Exercise 3.1
Let A and B be respectively the following DFAs:





(a) Compute the language partitions of A and B.

(b) Construct the quotients of A and B with respect to their language partitions.

(c) Give regular expressions for $L(A)$ and $L(B)$.

Exercise 3.2
A DFA $A = (Q, \Sigma, \delta, q_0, F)$ is *reversible* if no letter can enter a state from two distinct states, i.e. for every $p, q \in Q$ and $\sigma \in \Sigma$, if $\delta(p, \sigma) = \delta(q, \sigma)$, then $p = q$.

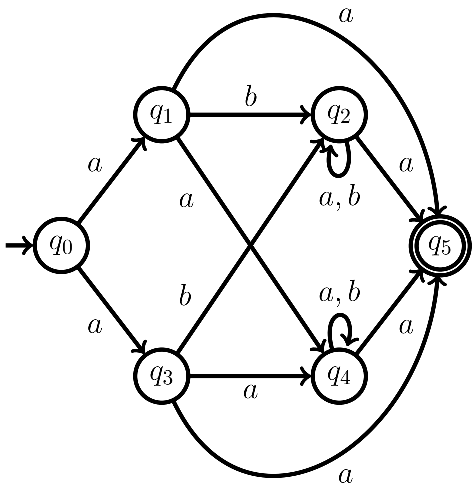
(a) Give a reversible DFA that accepts $L = \{ab, ba, bb\}$.

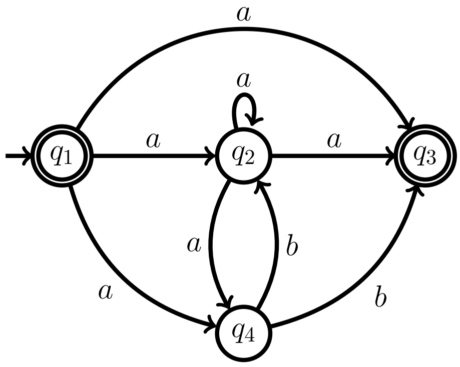
(b) Show that the minimal DFA accepting L is not reversible.

(c) Is there a unique minimal reversible DFA accepting L (up to isomorphism)? Justify your answer.

Exercise 3.3

Let A and B be respectively the following NFAs:





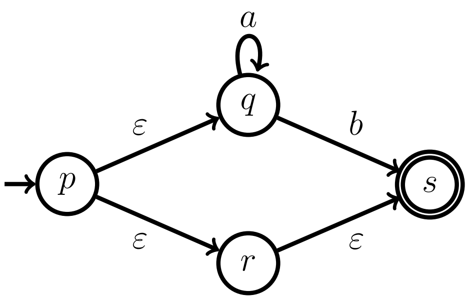
(a) Compute the coarsest stable refinements (CSR) of A and B.

(b) Construct the quotients of A and B with respect to their CSRs.

(c) Are the obtained automata minimal?

Exercise 3.4

Convert the following NFA-ε to an NFA using the algorithm *NFAεtoNFA* from the lecture notes (see Sect. 2.3, p. 31). You may verify your answer with the Python program `nfa-eps2nfa`.



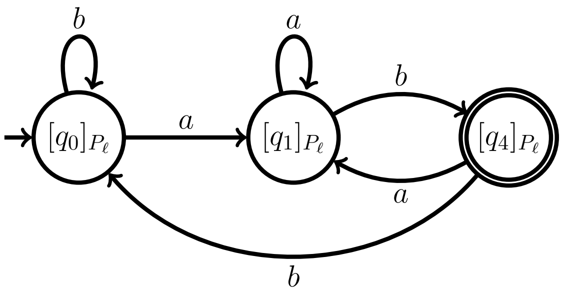
Solution 3.1

A) (a)

Iter.	Block to split	Splitter	New partition
0	—	—	$\{q_0, q_1, q_2, q_3, q_5, q_6\}, \{q_4\}$
1	$\{q_0, q_1, q_2, q_3, q_5, q_6\}$	$(b, \{q_4\})$	$\{q_0, q_2, q_6\}, \{q_1, q_3, q_5\}, \{q_4\}$
2	none, partition is stable	—	—

The language partition is $P_\ell = \{\{q_0, q_2, q_6\}, \{q_1, q_3, q_5\}, \{q_4\}\}$.

(b)



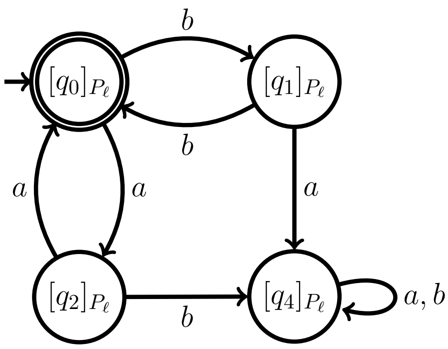
(c) $(a+b)^*ab$.

B) (a)

Iter.	Block to split	Splitter	New partition
0	—	—	$\{q_0, q_3\}, \{q_1, q_2, q_4\}$
1	$\{q_1, q_2, q_4\}$	$(b, \{q_1, q_2, q_4\})$	$\{q_0, q_3\}, \{q_1\}, \{q_2, q_4\}$
2	$\{q_2, q_4\}$	$(a, \{q_0, q_3\})$	$\{q_0, q_3\}, \{q_1\}, \{q_2\}, \{q_4\}$
3	none, partition is stable	—	—

The language partition is $P_\ell = \{\{q_0, q_3\}, \{q_1\}, \{q_2\}, \{q_4\}\}$.

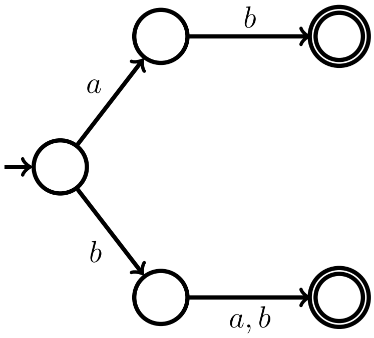
(b)



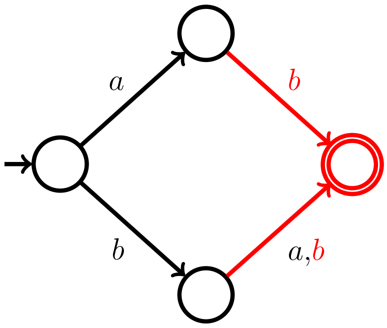
(c) $((aa)^*(bb)^*)^*$ or $(aa+bb)^*$.

Solution 3.2

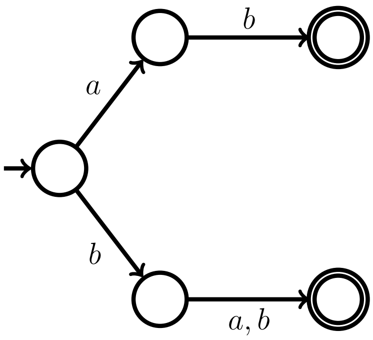
(a)

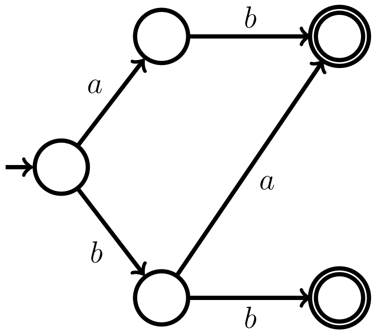


(b) By minimizing the previous reversible DFA, we obtain the following DFA which is not reversible since b enters the final state twice.



(c) No. The two following non isomorphic automata are both minimal reversible DFAs accepting L.





Note that, even though minimal reversible DFAs need not to be unique, testing whether a reversible DFA is minimal, and minimizing reversible DFAs can both be done in polynomial time. [1].

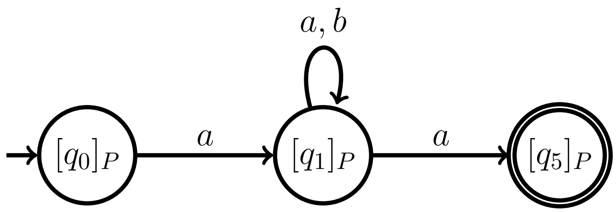
Solution 3.3

A) (a)

Iter.	**Block to split**	**Splitter**	**New partition**
0	—	—	$\{q_0, q_1, q_2, q_3, q_4\}, \{q_5\}$
1	$\{q_0, q_1, q_2, q_3, q_4\}$	$(a, \{q_5\})$	$\{q_0\}, \{q_1, q_2, q_3, q_4\}, \{q_5\}$
2	none, partition is stable	—	—

The CSR is $P = \{\{q_0\}, \{q_1, q_2, q_3, q_4\}, \{q_5\}\}$.

(b)



(c) Yes. The language accepted by the NFA is $L = a(a+b)^*a$. An NFA with one state can only accept $\emptyset, \{\varepsilon\}, a^*, b^*$ and $\{a,b\}^*$. Suppose there exists an NFA $A = (\{q_0, q_1\}, \{a, b\}, \delta, Q_0, F)$ accepting L. Without loss of generality, we may assume that q_0 is initial. A must respect the following properties:

- $q_0 \notin F$, since $\varepsilon \notin L$,
- $q_1 \in F$, since $L \neq \emptyset$,
- $q_1 \in \delta(q_0, a)$, otherwise it is impossible to accept aa which is in L.

This implies that A accepts a, yet $a \notin L$. Therefore, no two states NFA accepts L. $\square$

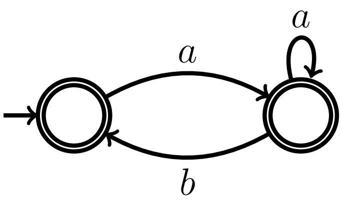
B) (a)

Iter.	**Block to split**	**Splitter**	**New partition**
0	—	—	$\{q_1, q_3\}, \{q_2, q_4\}$
1	$\{q_1, q_3\}$	$(a, \{q_2, q_4\})$	$\{q_1\}, \{q_2, q_4\}, \{q_3\}$
2	$\{q_2, q_4\}$	$(a, \{q_3\})$	$\{q_1\}, \{q_2\}, \{q_3\}, \{q_4\}$
3	none, partition is stable	—	—

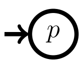
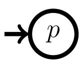
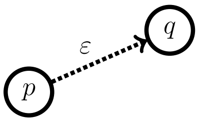
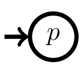
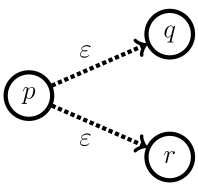
The CSR is $P = \{\{q_1\}, \{q_2\}, \{q_3\}, \{q_4\}\}$.

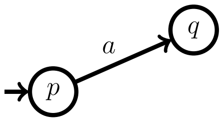
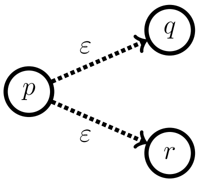
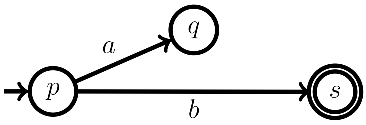
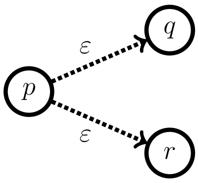
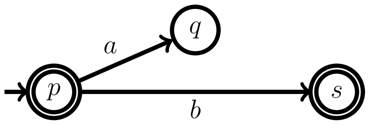
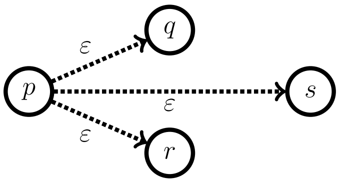
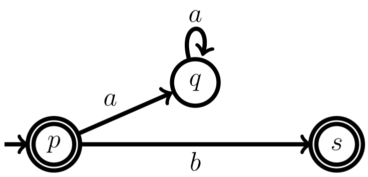
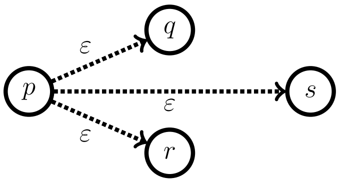
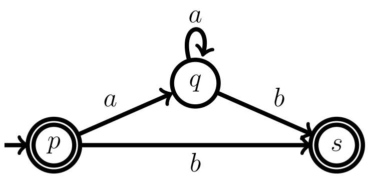
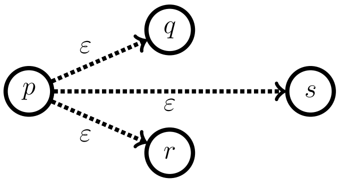
(b) The automaton remains unchanged.

(c) No. We have seen that this NFA accepts $(a + ab)^*$ in #2.1 of the second exercise sheet, and we have constructed a two states DFA for this language:

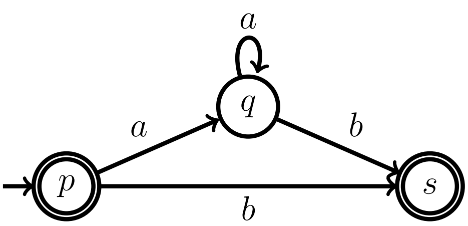


Solution 3.4

Iter.	$B = (Q', \Sigma, \delta', Q'_0, F')$	δ'' (**ε-transitions**)	**Workset** W **and next** (q_1, α, q_2)
0			$\{(p, \varepsilon, q), (p, \varepsilon, r)\}$
1			$\{(p, \varepsilon, r), (p, a, q), (p, b, s)\}$
2			$\{(p, a, q), (p, b, s), (p, \varepsilon, s)\}$

3			$\{(p,b,s),(p,\varepsilon,s),(q,a,q),(q,b,s)\}$
4			$\{(p,\varepsilon,s),(q,a,q),(q,b,s)\}$
5			$\{(q,a,q),(q,b,s)\}$
6			$\{(q,b,s)\}$
7			$\emptyset$

The NFA B obtained is:



which corresponds to the output of `nfa-eps2nfa`:

```
Q'  = set(['q', 'p', 's'])
S   = set(['a', 'b'])
d'  = set([('p', 'a', 'q'), ('q', 'b', 's'), ('q', 'a', 'q'), ('p', 'b', 's')])
Q0' = set(['p'])
F'  = set(['p', 's'])
```

References

[1] Markus Holzer, Sebastian Jakobi, and Martin Kutrib. Minimal reversible deterministic finite automata. In *Proc. 19th International Conference on Developments in Language Theory (DLT)*, pages 276–287, 2015.