Technische Universität München **Winter term 2015/16**
I7
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Automata and Formal Languages — Sample Solution 13

Due 22.01.2016

Solution 13.1

(a) Assuming that the DFS always prefer q_i over q_j whenever $i < j$, the sequence of the first DFS is $q_0, q_1, q_2, q_3, q_4, q_5, q_6$. Then the second DFS starts from q_6: $q_6, q_1, q_2, q_3, q_4, q_5, q_6$, in which point the algorithm reports NEMP.

(b) Only $q_0, q_1, q_3, q_4, q_6, q_1$.

(c) The run in (a) demonstrates the non-optimality of *NestedDFS*, because it does not return the lasso q_0, q_1, q_2, q_1.

(d) Assume the DFS choices as in (a). The DFS modifies the stacks as follows: $\mathbf{push}(q_0, C)$, $\mathbf{push}(q_0, V)$, $\mathbf{push}(q_1, C)$, $\mathbf{push}(q_1, V)$, $\mathbf{push}(q_2, C)$, $\mathbf{push}(q_2, V)$. Then, the algorithm inspects (q_2, q_0), and discovers that $q_0 \in V$, i.e. $q_0 \rightsquigarrow q_2$. It $\mathbf{pop}(C)$ twice, finds $q_1 \in F$, and reports NEMP.

(e) All three lassos:

- q_0, q_1, q_2, q_1
- $q_0, q_1, q_3, q_4, q_6, q_1$
- $q_0, q_1, q_3, q_4, q_5, q_1$

(f) The sequence of stack modifications until the algorithm reports NEMP is as follows:

$\mathbf{push}([q_0, \emptyset], C)$
$\mathbf{push}(q_0, V)$
$\mathbf{push}([q_1, \emptyset], C)$
$\mathbf{push}(q_1, V)$
$\mathbf{push}([q_2, \{0\}], C)$
$\mathbf{push}(q_2, V)$
$[q_2, \{0\}] \leftarrow \mathbf{pop}(C)$
$[q_1, \emptyset] \leftarrow \mathbf{pop}(C)$
$\mathbf{push}([q_1, \{0\}], C)$
$\mathbf{push}([q_3, \{1\}], C)$
$\mathbf{push}(q_3, V)$
$\mathbf{push}([q_4, \emptyset], C)$
$\mathbf{push}(q_4, V)$
$\mathbf{push}([q_5, \{1\}], C)$
$\mathbf{push}(q_5, V)$
$[q_5, \{1\}] \leftarrow \mathbf{pop}(C)$
$[q_4, \emptyset] \leftarrow \mathbf{pop}(C)$
$[q_3, \{1\}] \leftarrow \mathbf{pop}(C)$
$[q_1, \{0\}] \leftarrow \mathbf{pop}(C)$

Solution 13.2

Input: Weak NBA $A = (Q, \Sigma, \delta, q_0, F)$
Output: EMP if $L_\omega(A) = \emptyset$, NEMP otherwise

```
1 S, V ← ∅
2 dfs(q_0)
3 report EMP
4 proc dfs(q)
5 |   add q to S; add q to V
6 |   for all r ∈ δ(q) do
7 |   |   if r ∉ S then dfs(r)
8 |   |   else if r ∈ V and r ∈ F then report NEMP
9 |   remove q from V
```