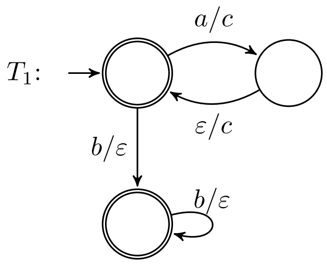
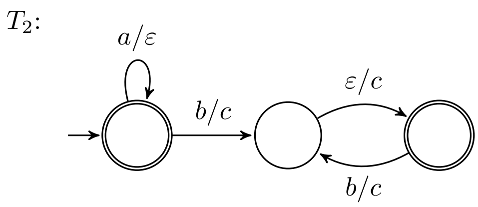
Automata and Formal Languages — Sample Solution 7

Due 27.11.2015

Solution 7.1

(a) The following figures use the notation a/b for $(a, b) \in (\Sigma \cup \{\varepsilon\}) \times (\Sigma \cup \{\varepsilon\})$.

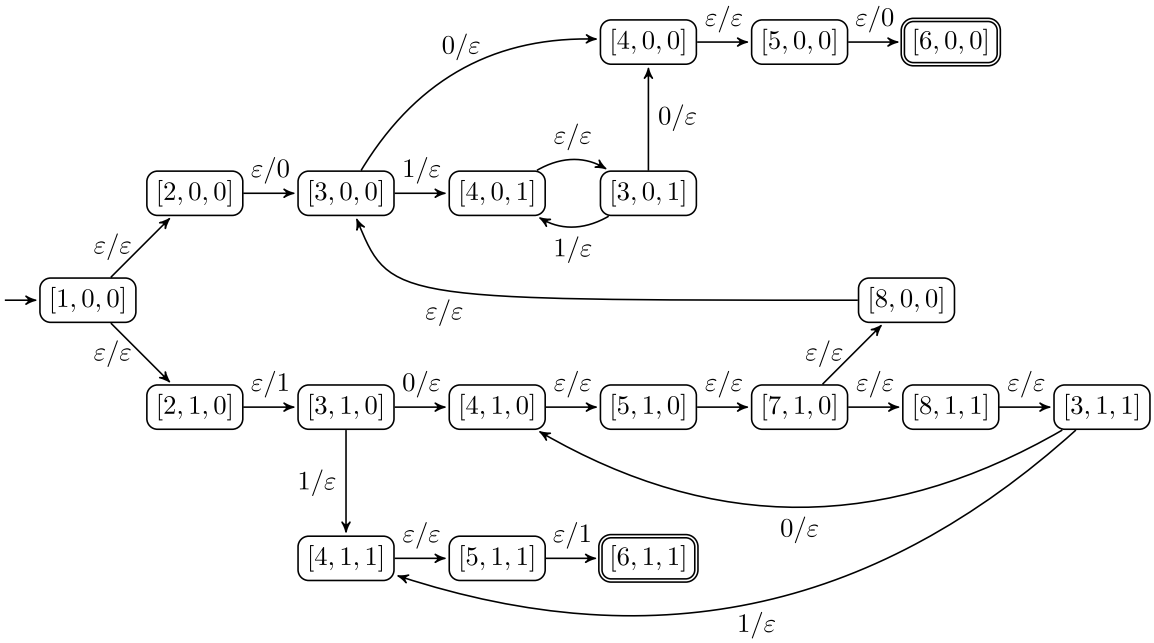




(b) $L(T_1) \cap L(T_2) = \{(a^n b^n, c^n) \mid n \geq 0\}$. Assume that there is a transducer T that recognizes $L(T_1) \cap L(T_2)$. Replace each transition $q \overset{a/c}{\to} q'$ by $q \overset{a}{\to} q'$ and $q \overset{b/c}{\to} q'$ by $q \overset{b}{\to} q'$. The resulting NFA recognizes $\{a^n b^n \mid n \geq 0\}$, a contradiction.

Solution 7.2

(a) Transducer T:



(b) No.

(c) $0, 1$

(d) No.

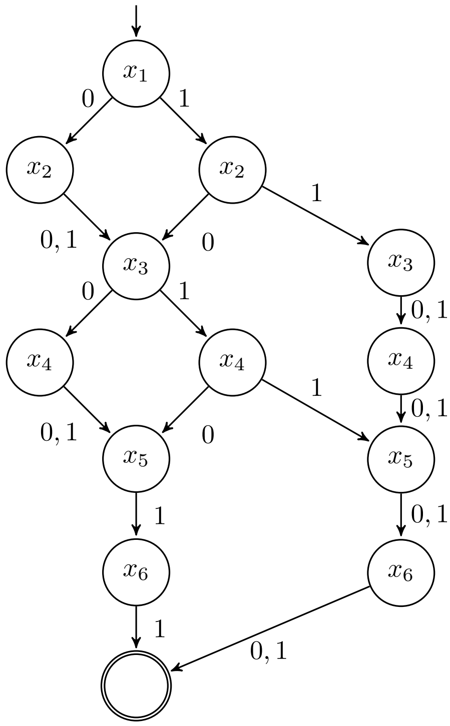
(e) Let A_I and A_O be NFAs for I and O, respectively. A possible algorithm is to compute:

$$\mathit{EmptyNFA}(\mathit{IntersNFA}(\mathit{Post}^{\varepsilon}(A_I, T), A_O)) \ .$$

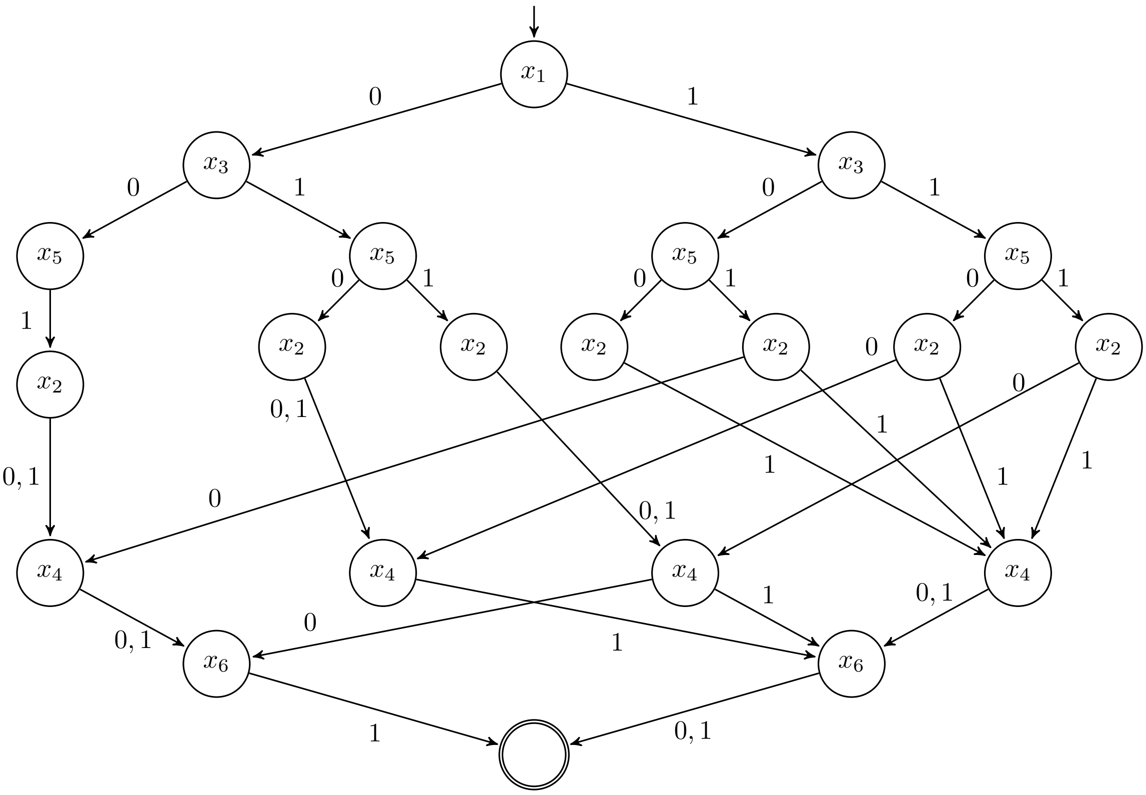
Note that $\mathit{Post}^{\varepsilon}$ is similar to Post in the lecture. A slight modification is neeeded because T is an ε-transducer.

Solution 7.3

(a)



(b)



(c) We can generalize observations of the above automata. For the first language, we need at most 5 states for each pair $x_{2k-1} \cdot x_{2k}$. Therefore, the minimal DFA has $O(n)$ states.

For the second language, observe that every assignment of $x_1, \ldots, x_{2n-1}$ results in a different assignment of $x_2, \ldots, x_{2n}$. We need a different state for every different word of length n, and therefore the minimal DFA has at least 2^n states.