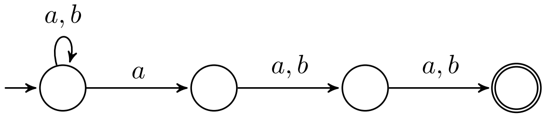
Technische Universität München **Winter term 2015/16**
I7
Prof. J. Esparza / Dr. D. Suwimonteerabuth / Dr. C. Broadbent **14.10.2015**

Automata and Formal Languages — Sample Solution 1

Due 21.10.2015

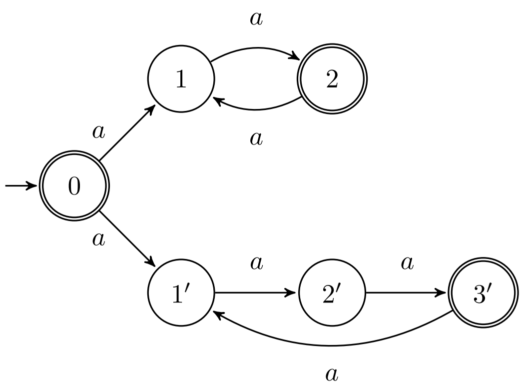
Solution 1.1

(a) NFA for C_3:



(b) $n+1$ states

(c) NFA for L_3:



(d) 7 states

(e) We need to prove that $\varepsilon, a, aa, \ldots, a^{n(n-1)-1}$ all belong to different residuals, hence there are at least $n(n-1)$ residuals. Consider two arbitrary a^x and a^{x+d}, where $0 \leq x < x+d < n(n-1)$. We need to find $z \in \mathbb{N}$ such that L_n accepts either $a^x a^z$ or $a^{x+d}a^z$, but not both. We consider four possible cases:

Case 1 Neither n nor $n-1$ divides d. Let $z = n(n-1) - x$. We have $a^x a^z = a^{n(n-1)} \in L_n$. On the other hand, $a^{x+d}a^z = a^{n(n+1)+d} \notin L_n$, because neither n nor $n-1$ dividees d.

Case 2 n divides d, but $n-1$ does not. Let $z = n(n-1) - x - (d \mod (n-1))$. We have $a^x a^z = a^{n(n-1)-(d \mod (n-1))} \notin L_n$, because $d \mod (n-1) < n-1 < n$. On the other hand, $a^{x+d}a^z = a^{n(n+1)+d-(d \mod (n-1))} \in L_n$, because $n-1$ dividees $d - (d \mod (n-1))$.

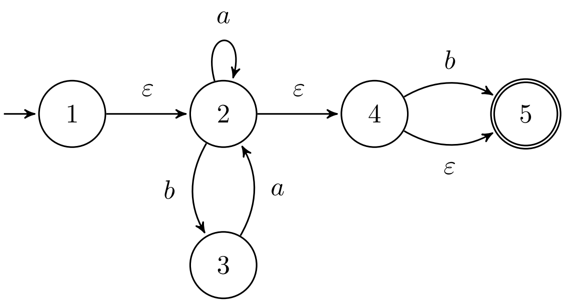
Case 3 $n-1$ divides d, but n does not. Similar to Case 2.

Case 4 Both $n-1$ and n devide d. This cannot happen, because $1 \leq d < n(n-1) = \mathrm{lcm}(n, n-1)$.

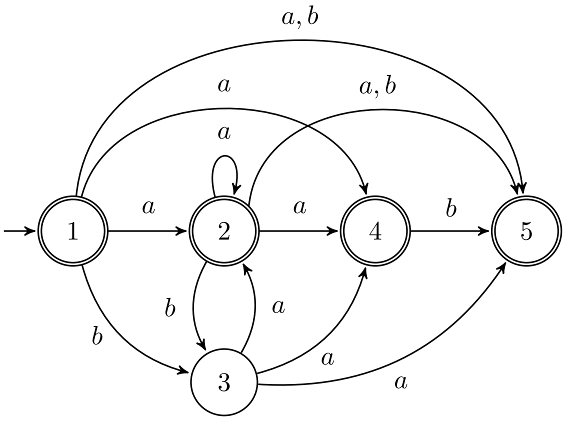
Solution 1.3

(a) RE: $(a+ba)^*(b+\varepsilon)$

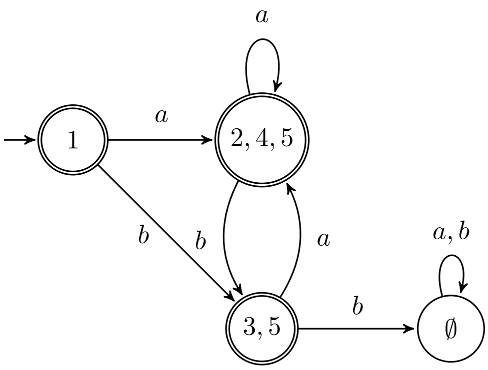
NFA-ε:



NFA:



DFA:



Solution 1.4

Let $A = (Q_A, \Sigma_A, \delta_A, q_{0,A}, F_A)$ and $B = (Q_B, \Sigma_B, \delta_B, q_{0,B}, F_B)$ be DFAs accepting L_1 and L_2, respectively. We define $C = (Q, \Sigma, \delta, q_0, F)$, where $Q = Q_A \times Q_B$, $\Sigma = \Sigma_A \cup \Sigma_B$, $q_0 = (q_{0,A}, q_{0,B})$, $F = F_A \times F_B$, and for every $q_A \in Q_A$, $q_B \in Q_B$, and $a \in \Sigma$:

$$\delta((q_A, q_B), a) = \{(\delta_A(q_A, a), q_B)\} \cup \{(q_A, \delta_B(q_B, a))\}\ .$$

Obviously, C accepts $S(L_1, L_2)$.