Technische Universität München
Winter term 2015/16
Prof. J. Esparza / J. Křetínský *et al.*

Name:

Matrikelnummer:

Automata and Formal Languages — Endterm

Answer all questions. Question 1 requires only short answers that can be written directly on this sheet. All other questions should be answered in a separate booklet. A total of 40 points is available.

Question 1

(a) Prove (via a proof) or disprove (via a counterexample) the following LTL equivalence.

$$(\mathbf{G}p)\ \mathbf{U}\ (\mathbf{G}q) \qquad \equiv \qquad \mathbf{G}(p\ \mathbf{U}\ q)$$

Answer: ______________________________

[1 point]

(b) Let $\Sigma = \{a, b\}$. Give formulae of $MSO(\Sigma)$ for each of the following two languages. You may use the following macros from the lecture notes:

$$\begin{aligned}
\text{first}(x) &:= \neg\exists y\ y < x \\
y = x+1 &:= x < y \wedge \neg\exists z\ (x < z \wedge z < y) \\
y = x+2 &:= \exists z\ (z = x+1 \wedge y = z+1) \\
X \subseteq Y &:= \forall x\ (x \in X \rightarrow x \in Y) \\
\text{Odd}(X) &:= \forall x\ (x \in X \leftrightarrow (\text{first}(x) \vee \exists z\ (x = z+2 \wedge z \in X)))
\end{aligned}$$

(i) ab^*

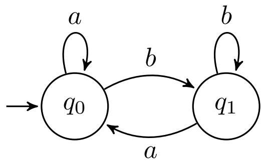
Answer: ______________________________

(ii) $(ab+ba)^*$

Answer: ______________________________

[2 points]

(c) Consider the following Müller automaton with $\mathcal{F} = \{\{q_0, q_1\}, \{q_1\}\}$:



(i) Describe the language recognized by the automaton.

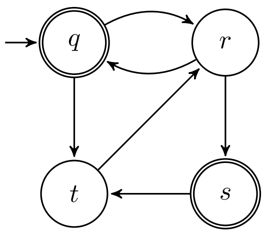
Answer: ______________________________

(ii) Construct an equivalent *deterministic* Büchi automaton.

Answer: ______________________________

[2 points]

(d) Consider the following Büchi automaton:



Which lassos can be reported by the *NestedDFS* algorithm?

Answer: ______________________________

[2 points]

(e) Draw a *deterministic* co-Büchi automaton recognising the language $(a + b)^*(aa + bb)^\omega$.

Answer: ______________________________

[2 points]

Name:

Matrikelnummer:

(f) Fill in the gaps in the following algorithm (in lines 2, 7 and 11) so that it meets its specification.

Input: DFAs $A_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$, $A_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$
Output: DFA $A = (Q, \Sigma, \delta, q_0, F)$ with $L(A) = (L(A_1) \cap L(A_2)) \cup \overline{L(A_2)}$

1 $Q, \delta, F \leftarrow \emptyset$

2 $q_0 \leftarrow$ ______________________________

3 $W \leftarrow \{q_0\}$

4 **while** $W \neq \emptyset$ **do**

5   **pick** $[q_1, q_2]$ **from** W

6   **add** $[q_1, q_2]$ **to** Q

7   **if** ____ **then** __________________________

8   **for all** $a \in \Sigma$ **do**

9         $q_1' \leftarrow \delta_1(q_1, a)$; $q_2' \leftarrow \delta_2(q_2, a)$

10   **if** $[q_1', q_2'] \notin Q$ **then add** $[q_1', q_2']$ **to** W

11   **add** ____ **to** δ

[1 point]

(g) Draw a Büchi automaton recognising the infinite sequences σ of subsets of $\{ p, q \}$ such that

$$\sigma \models \mathbf{G}(\mathbf{F}(p \ \mathbf{U} \ q))$$

(You do *not* need to use the conversion algorithm from the lectures).

Answer: ____

[2 points]

[Total: 12 points]

This page is intentionally left blank.

Question 2

Given two languages L_1 and L_2 over an alphabet Σ, define the language L_{12} as follows:

$$w \in L_{12} \quad \text{iff} \quad \text{there is a word } u_1u_2 \in L_1 \text{ and a word } v \in L_2 \text{ such that } w = u_1v.$$

Show that if L_1 and L_2 are regular, then L_{12} is regular.

[Total: 4 points]

Question 3

(a) Draw a DFA recognising $(a + bb)^*$.

(b) Given a language $L \subseteq \Sigma^*$ and $w \in \Sigma^*$, give a formal definition of the residual L^w.

(c) Give regular expressions for every residual of the language $(a + bb)^*$.

(d) Prove that the language $L' := \{ a^{2^n} \mid n \in \mathbb{N}_0 \}$ has infinitely many residuals. Is L' regular? Justify your answer.

[Total: 6 points]

Question 4

For this question, use the LSBF encoding of the natural numbers $\mathbb{N}_0$.

(i) Given a binary relation $R \subseteq \mathbb{N}_0 \times \mathbb{N}_0$, what does it mean to say that R is regular?

(ii) Show that the relation $R_1 := \{ (n, 2n) \mid n \in \mathbb{N}_0 \}$ is regular.

(iii) Show that the relation $R_2 := \{ (n, 1 + n) \mid n \in \mathbb{N}_0 \}$ is regular.

(iv) You may assume that given two regular relations P_1 and P_2, their composition

$$P_1 \circ P_2 := \{ (n, m) \mid (n, k) \in P_1 \text{ and } (k, m) \in P_2 \text{ for some } k \in \mathbb{N}_0 \}$$

is also regular. Use this fact together with the facts that R_1 and R_2 from parts (ii) and (iii) are regular to show that the following relation is regular

$$R := \{ (n, 4n + 3) \mid n \in \mathbb{N}_0 \}$$

[Total: 6 points]

Question 5

Construct a LazyDFA recognising words over the alphabet $\{a, b, c\}$ that contain the pattern *abacab*. **[Total: 3 points]**

Question 6

Consider the following program with two parallel processes. The domain of x is $\{0, 1\}$, and the initial value is 0.

Process 1:

```
while true do
    if x = 0 then
        x ← 1
```

Process 2:

```
while true do
    x ← 0
```

(a) Model the program by constructing a network of three automata (one for each process and one for the variable x) and then drawing their asynchronous product.

(b) Consider the set of atomic propositions $AP = \{x = 0, x = 1\}$. Give an LTL formula for the following property: $x = 0$ holds infinitely often. Construct a Büchi automaton for the *negation* of this property.

(c) Does the program satisfy the property in (b)? Why?

[Total: 5 points]

Question 7 ⋆

(a) Consider the language $L := \{ 0^i \mid i \text{ is prime } \}$ (which is not regular). Prove that $L^* = \epsilon + 000^*$.

(b) Now let L be an *arbitrary* language over the single-letter alphabet $\{ 0 \}$. (L is not necessarily regular). Prove that L^* is regular.

[Total: 4 points]

End of Examination