Technische Universität München **Winter term 2015/16**
I7
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Automata and Formal Languages — Homework 15

Due 01.02.2016

Exercise 15.1
Let $\Sigma = \{a, b\}$. Determine the mapping $\mathcal{J}$ for each of the following $\varphi \in MSO(\Sigma)$ such that

$$(w, \mathcal{J}) \models \varphi$$

for all $w \in L_\omega(((a+b)a)^\omega)$.

(a) $\forall x \in X\ Q_a(x)$

(b) $x \in X \wedge y \in X \wedge x < y \wedge Q_a(x) \wedge Q_a(y)$

(c) $\forall x \in X\ \forall y \in Y\ \ (x < y \rightarrow \exists z\ (x < z \wedge z < y \wedge Q_a(z)))$

Exercise 15.2
Let $\Sigma = \{a, b, c\}$. Give formulae of $MSO(\Sigma)$ for the following ω-languages:

(a) ab^ω

(b) $(a+b)^\omega$

(c) $(ab)^\omega$

(d) $(a^*b)^\omega$

(e) $(a^*b)^\omega \vee (a^*b)^*a^\omega$

(f) The set of ω-words in which every even position is a

(g) The set of ω-words without subwords aaa

(h) The set of ω-words without subwords ac^*ac^*a

(i) The set of ω-words with finitely many a's

(j) The set of ω-words with infinitely many a's

Exercise 15.3
Assume that there exists an algorithm *RegToMSO* that accepts a regular expression r as input and output a sentence φ such that $L(\varphi) = L(r)$.

Give an algorithm ω-*RegToMSO* that accepts an ω-regular expression s as input and directly constructs a sentence ψ such that $L(\psi) = L_\omega(s)$, without constructing any automata.

Hint: Break down the structure of s and construct the sentence bottom-up. Think about how to construct a formula φ^X from a formula φ of $MSO(\Sigma)$ and a free second-order variable X expressing "the projection of the word onto the positions of X satisfies φ". Formally, for every mapping $\mathcal{J}$ of φ^X we have $(w, \mathcal{J}) \models \varphi^X$ iff $(w|_{\mathcal{J}(X)}, \mathcal{J})$, where $w|_{\mathcal{J}(X)}$ denotes the result of deleting from w the letters at all positions that do not belong to $\mathcal{J}(X)$.