Technische Universität München **Winter term 2015/16**
I7
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Automata and Formal Languages — Homework 9

Due 11.12.2015

Exercise 9.1
Let $\Sigma = \{a, b\}$. Give formulae of $FO(\Sigma)$ for the following languages:

(a) $(a+b)(a+b)^*$

(b) $a(a+b)^*$

(c) $(aa^*b)^*$

(d) a^*b^*

(e) $L_n = \{ww \in \Sigma^* \mid |w| = n\}$ where $n \geq 1$.

(f) $L'_n = \{ww \in \Sigma^* \mid |w| = 2^n\}$ where $n \geq 1$. The formula must be of polynomial length in n. Hint: construct first an abbreviation of polynomial length for $y = x + 2^n$.

Exercise 9.2
Let $\Sigma = \{a, b\}$. Give formulae of $MSO(\Sigma)$ for the following languages:

(a) The set of words in which every odd position is a.

(b) The set of words of odd length containing only a's.

(c) The set of words with an odd number of occurences of a's.

(d) The set of words of odd length with an odd number of occurences of a's.

(e) The set of words, where between every two b's there is a block of odd number of occurences of a's.

Exercise 9.3
Consider the logic *Pure_MSO*(Σ) with syntax

$$\varphi := X \subseteq Q_a \mid X < Y \mid X \subseteq Y \mid \neg\varphi \mid \varphi \vee \varphi \mid \exists X\ \varphi$$

Notice that formulas of *Pure_MSO*(Σ) do not contain first-order variables. The satisfaction relation of *Pure_MSO*(Σ) is given by:

$$\begin{array}{lcll} (w, \mathcal{I}) & \models & X \subseteq Q_a & \text{iff} \quad w[p] = a \text{ for every } p \in \mathcal{I}(X) \\ (w, \mathcal{I}) & \models & X < Y & \text{iff} \quad p < p' \text{ for every } p \in \mathcal{I}(X),\ p' \in \mathcal{I}(Y) \\ (w, \mathcal{I}) & \models & X \subseteq Y & \text{iff} \quad \mathcal{I}(X) \subseteq \mathcal{I}(Y) \end{array}$$

with the rest as for $MSO(\Sigma)$.

Prove that $MSO(\Sigma)$ and *Pure_MSO*(Σ) have the same expressive power for sentences. That is, show that for every sentence ϕ of $MSO(\Sigma)$ there is an equivalent sentence ψ of *Pure_MSO*(Σ), and vice versa.