Technische Universität München **Winter term 2015/16**
I7
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Automata and Formal Languages — Homework 1

Due 21.10.2015

Exercise 1.1
Go to `http://www.jflap.org/` and download JFLAP. Run it and select the finite automata mode.

(a) Consider the language $C_n = \Sigma^* a \Sigma^{n-1}$ over $\Sigma = \{a, b\}$. Draw an NFA that recognizes C_3 and determinize it using JFLAP.

(b) Consider a similar language $D_n = \Sigma^* a (\Sigma \cup \varepsilon)^{n-1}$ over $\Sigma = \{a, b\}$. At least how many states does a DFA require to recognize D_n? Justify your answer.

(c) Let $L_n = \{a^k \mid k \text{ is divisible by } n \text{ or } n-1\}$ be a language over $\Sigma = \{a\}$. Draw an NFA A that recognizes L_3.

(d) Use JFLAP to determinize A. How many states does A have?

(e) Show that every DFA recognizing L_n has at least $n(n-1)$ states.

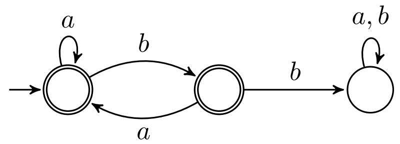
Exercise 1.2
Download a conversion game from `https://www7.in.tum.de/tools/jflap-game/`. Select the coversion game mode to play the game. Finish the following conversion types:

(a) Guess DFA from NFA, RE

(b) Guess NFA from RE

(c) Guess RE from DFA, NFA

Exercise 1.3
Let A be the following automaton:



(a) Transform the automaton A into an equivalent regular expression, then transform this expression into an NFA (with ε-transitions), remove the ε-transitions, and determinize the automaton.

(b) Use JFLAP to perform the same transformations. Is there any difference?

(c) Use JFLAP to check that your resulting automaton is equivalent to the original one.

Exercise 1.4
Given an alphabet Σ, we say that w is a *shuffle* of words u and v, if there exist $u_i, v_i \in \Sigma^*$ such that $u = u_1 \cdots u_k$, $v = v_1 \cdots v_k$, and $w = u_1 v_1 \cdots u_k v_k$.

Given languages L_1 and L_2, we define the shuffle of L_1 and L_2 as

$$S(L_1, L_2) = \{w \mid \exists u \in L_1, v \in L_2 \text{ s.t. } w \text{ is a shuffle of } u \text{ and } v\}$$

Show that if L_1 and L_2 are regular, then $S(L_1, L_2)$ is also regular.