Technische Universität München
Prof. M. Kufleitner / A. Durand-Gasselin
Winter term 2013/2014

Name:
Matrikelnummer:
Signature:

Automata and Formal Languages – Endterm

- If you feel ill, let us know immediately.
- Please, **do not write** until told so. You are given approx. 10 minutes to read the exercises and address us in case of questions or problems.
- You will be given **120 minutes** to fill in all the required information and write down your solutions.
- You can use your **own paper** for writing down the solutions.
- Should you require **paper**, please tell us.
- **Sign** the exercise sheet (upper right corner) and hand it back with the solutions.
- Write your **name and Matrikelnummer** on every paper.
- Write with a non-erasable **pen**, do not use red or green color.
- You are **not allowed to use auxiliary means** other than your pen and paper.
- You may answer in **English or German**.
- Please turn off your **cell phone**.
- You can obtain **40 points** in the exam. You need **17 points** in total to pass.
- Good luck!

Throughout, let $\Sigma = \{a, b\}$.

Exercise 1 **(6×2P = 12P)**

Let $L = (a\{ab, ba\}^*b)^*$.

(a) Use the Thompson construction for finding a nondeterministic automaton $\mathcal{A}$ with $L(\mathcal{A}) = L$.

(b) Use removal of ε-transitions for constructing an equivalent nondeterministic automaton $\mathcal{B}$ for Σ.

(c) Use the powerset construction for finding an equivalent deterministic automaton $\mathcal{C}$.

(d) Use a minimization algorithm for constructing the minimal automaton $\mathcal{A}_L$.

(e) Give an automaton $\mathcal{D}$ with $L(\mathcal{D}) = \Sigma^* \setminus L$.

(f) Use state elimination for constructing a rational expression for $\Sigma^* \setminus L$.

Exercise 2 **(4P)**

Apply Angluin's L^*-algorithm for learning the language $\{aa, b\}^*$ over the alphabet Σ. Start with samples $S = \{\varepsilon\}$ and extensions $E = \{\varepsilon\}$ and use counterexamples of minimal length.

Exercise 3 **(2P)**

Give an MSO sentence defining the language $\{aa, ab\}^*$ over the alphabet Σ.

Exercise 4 **(3P)**

We use binary encoding of nonnegative integers with the least significant bit first. Give a finite automaton accepting the solutions of $2x > y + z$.

Exercise 5 **(4P)**

Let $\mathcal{A} = (Q, \delta, I, F)$ with $\delta \subseteq Q \times \Sigma \times Q$ be a nondeterministic finite Σ^*-automaton. Consider the following decision problem:

Input: A binary encoded integer $n \geq 0$.

Question: Is $(ab)^n \in L(\mathcal{A})$?

Show that this problem can be solved in deterministic polynomial time.

Exercise 6 **(2P + 2P = 4P)**

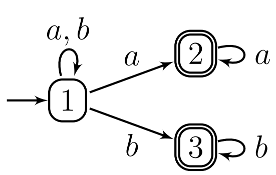
Let $L = \{\alpha \in \Sigma^\omega \mid \alpha$ has only finitely many occurrences of ab as a factor$\}$.

(a) Give a Büchi automaton which accepts L.

(b) Show that there is no finite deterministic Büchi automaton which accepts L.

Exercise 7 **(5P)**

Let $\mathcal{A}$ be the following Büchi automaton over Σ:



Apply Safra's construction to give a determinisitc Rabin automaton for $L(\mathcal{A})$.

Exercise 8 **(3P + 1P + 2P = 6P)**

Let $S = \{a, b\}$ be the semigroup defined by $a = a^2 = ab$ and $b = b^2 = ba$, and let the homomorphism $h : \Sigma^+ \to S$ be defined by $h(a) = a$ and $h(b) = b$. For $s \in S$ we use the notation $[s]$ instead of $h^{-1}(s)$.

(a) Give a Büchi automaton for the language $[a][b]^\omega$.

(b) When is a language $L \subseteq \Sigma^\omega$ recognized by a homomorphism $g : \Sigma^+ \to T$ for some finite semigroup T? Give the definition or some equivalent condition.

(c) Is $[a][b]^\omega$ recognized by h? Justify your answer.