Technische Universität München **Winter term 2013/2014**
I7
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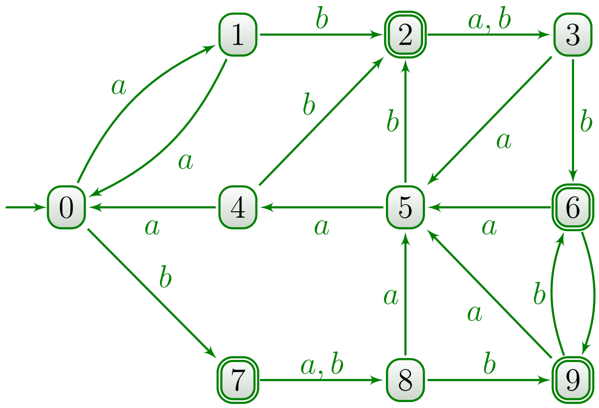
Automata and Formal Languages – Exercise sheet 8

Exercise 8.1

Apply Hopcroft's minimization algorithm to the following automaton:



Exercise 8.2

Let $S = \{a, b, ab\}$ be the semigroup defined by $a^2 = b \cdot a = a$ and $b^2 = b$, let $\Sigma = \{a, b\}$, and let the homomorphism $h : \Sigma^+ \to S$ be defined by $h(a) = a$ and $h(b) = b$. For $s \in S$ we use the notation $[s]$ instead of $h^{-1}(s)$. Consider $\alpha, \beta \in \Sigma^\omega$ with $\alpha, \beta \in [s][t]^\omega$ for some $s, t \in S$ and let $s', t' \in S$ be arbitrary. Which of the following claims hold and why?

(a) If $\alpha \in [s'][t']^\omega$, then $\beta \in [s'][t']^\omega$.

(b) If $\alpha \in [s][t']^\omega$, then $\beta \in [s][t']^\omega$.

(c) If $\alpha \in [s'][t]^\omega$, then $\beta \in [s'][t]^\omega$.

Exercise 8.3

Let $h : \Sigma^+ \to \Sigma^+$ a homorphism such that $h(a) = au$ for some $u \in \Sigma^+$.

(a) Show that $h^\omega(a) := \lim_{n\to\infty} h^n(a)$ defines an ω-word.

(b) What is $h^\omega(a)$ when $\Sigma = \{a, b, c\}$, $h(a) = ab$, $h(b) = ccb$, $h(c) = c$?

(c) Let $g : \Sigma^+ \to \Sigma^+$ be a homomorphism and $\mathcal{A}$ a Büchi automaton. Can we decide whether $g(h^\omega(a)) \in L(\mathcal{A})$?