Technische Universität München **Winter term 2013/2014**
I7
M. Kufleitner / A. Durand-Gasselin **9.01.2014**

Automata and Formal Languages – Exercise sheet 7

Exercise 7.1

Let $L \subseteq \Sigma^*$ and let $\$ \notin \Sigma$. Show that L is regular if and only if $L\$^\omega \subseteq (\Sigma \cup \{\$\})^\omega$ is ω-regular.

Exercise 7.2

Let $\mathcal{A} = (Q, \delta, I, F)$ be an automaton with $\delta \subseteq Q \times \Sigma \times Q$. Let $L_{\text{fin}} \subseteq \Sigma^*$ be the language of finite words accepted by the nondeterministic automaton $\mathcal{A}$ and let $L(\mathcal{A}) \subseteq \Sigma^\omega$ be the language accepted by the Büchi automaton $\mathcal{A}$.

(a) Give automata $\mathcal{A}_1, \mathcal{A}_2$ such that $L_{\text{fin}}(\mathcal{A}_1) = L_{\text{fin}}(\mathcal{A}_2)$ and $L(\mathcal{A}_1) \neq L(\mathcal{A}_2)$.

(b) Give deterministic automata $\mathcal{A}_1, \mathcal{A}_2$ with $L_{\text{fin}}(\mathcal{A}_1) \neq L_{\text{fin}}(\mathcal{A}_2)$ and $L(\mathcal{A}_1) = L(\mathcal{A}_2)$.

(c) Show that if $\mathcal{A}_1, \mathcal{A}_2$ are deterministic with $L_{\text{fin}}(\mathcal{A}_1) = L_{\text{fin}}(\mathcal{A}_2)$, then $L(\mathcal{A}_1) = L(\mathcal{A}_2)$.

Exercise 7.3

Let $\Sigma = \{a, b\}$.

(a) Give Büchi automata for the following languages.

- $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ has infinitely many } a\text{'s}\}$.
- $L_2 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ has only finitely many } b\text{'s}\}$.
- $L_3 = \{\alpha \in \Sigma^\omega \mid \text{every } a \text{ in } \alpha \text{ is followed by } b\}$.

(b) Give an automaton for $L_1 \cap L_2 \cap L_3$ using the construction from the lecture.

(c) Give an automaton for $\Sigma^\omega \setminus L_1$ using the construction from the lecture.