Technische Universität München **Winter term 2013/2014**
I7
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Automata and Formal Languages – Exercise sheet 4

Exercise 4.1

Let $\Sigma = \{a, b\}$. We consider the languages $L_1 = (baa^*)^*b$ and $L_2 = \Sigma^*a\Sigma^2a\Sigma^*$.

(a) Use the Thompson-construction for finding nondeterministic automata $\mathcal{A}_1$ and $\mathcal{A}_2$ with $L_i = L(\mathcal{A}_i)$.

(b) Use removal of ε-transitions for constructing equivalent nondeterministic automata $\mathcal{B}_1$ and $\mathcal{B}_2$ with labels in Σ.

(c) Use Brzozowski's minimization algorithm for constructing the minimal automata for L_1 and L_2.

(d) Build an automaton $\mathcal{D}$ accepting $L_1 \setminus L_2$ and minimize it.

(e) Use state elimination for constructing a rational expression for $L_1 \setminus L_2$.

Exercise 4.2

We use binary encoding of nonnegative integers with the least significant bit first.

(a) Give an automaton accepting the solutions of $3x - 5y > 2$.

(b) Give an automaton accepting the solutions of the formula $x + 2y \equiv 0 \bmod 7$.

(c) Let $a_1, \ldots, a_k \in \mathbb{Z}$ and $c \in \mathbb{N}$. Give an algorithm to build the automaton accepting solutions of $\sum_{i=1}^{k} a_i x_i = c$.

(d) Use the automata theoretic approach for showing that $\forall x \exists y \exists z\colon -x + 2y + 3z = 2$ is true for $\mathbb{N}$.

Exercise 4.3

Show that the true sentences in $\mathrm{FO}[\mathbb{Z}, +]$ are decidable, i.e., for any given first-order sentence φ using atomic formulas of the form $x = n$ for $n \in \mathbb{Z}$ and $x + y = z$, one can decide whether φ is true over the integers $\mathbb{Z}$. For instance, the sentence $\forall x \exists y \exists z\colon 3y + 5z = x$ is false over $\mathbb{N}$ but true over $\mathbb{Z}$.