Technische Universität München **Winter term 2013/2014**
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Automata and Formal Languages – Exercise sheet 3

Exercise 3.1

A word $u = a_1 \cdots a_n$ is a *scattered subword* of w (written as $u \lhd w$) if $w = w_0 a_1 w_1 \cdots a_n w_n$ for some $w_0, \ldots, w_n \in \Sigma^*$. Let $L \subseteq \Sigma^*$ be regular.

(a) Show that $L_\rhd = \{u \in \Sigma^* \mid w \lhd u \text{ for some } w \in L\}$ is also regular.

(b) Show that $L_\lhd = \{u \in \Sigma^* \mid u \lhd w \text{ for some } w \in L\}$ is also regular.

Exercise 3.2

Let $\Sigma = \{a, b\}$ and $L = ((ab)^* a \cup abb)^* \in \mathrm{RAT}(\Sigma^*)$.

(a) Use the Thompson-construction for finding a nondeterministic automaton $\mathcal{A}$ with $L = L(\mathcal{A})$.

(b) Use removal of ε-transitions for constructing an equivalent nondeterministic automaton $\mathcal{B}$ for Σ.

(c) Use the power set construction for finding an equivalent deterministic automaton $\mathcal{C}$.

(d) Use Moore's minimization algorithm for constructing the minimal automaton $\mathcal{A}_L$.

(e) Give an automaton $\mathcal{D}$ with $L(\mathcal{D}) = \Sigma^* \setminus L$.

(f) Use state elimination for constructing a rational expression for $\Sigma^* \setminus L$.

Exercise 3.3

Let M_1 and M_2 be monoids. The direct product of M_1 and M_2 is $M_1 \times M_2$ equipped with the operation $(u_1, u_2) \cdot (v_1, v_2) = (u_1 v_1, u_2 v_2)$. Show that $L \in \mathrm{REC}(M_1 \times M_2)$ if and only if L is a finite union of sets $K_1 \times K_2$ with $K_1 \in \mathrm{REC}(M_1)$ and $K_2 \in \mathrm{REC}(M_2)$.

Exercise 3.4

Give a language $L \in \mathrm{RAT}(a^* \times b^*)$ such that L cannot be written as a finite union of languages $K_1 \times K_2$ with $K_1 \in \mathrm{RAT}(a^*)$ and $K_2 \in \mathrm{RAT}(b^*)$