Technische Universität München **Winter term 2013/2014**
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Automata and Formal Languages – Exercise sheet 2

Exercise 2.1

Let $L = \{ab, abb\}^*$. Give the minimal automata of $K_a = \{u \in \{a,b\}^* \mid u \equiv_L a\}$ and $K_b = \{u \in \{a,b\}^* \mid u \equiv_L b\}$.

Exercise 2.2

Let M be generated by $\Sigma \subseteq M$, let $\varphi : M \to N$ be a homomorphism to a finite monoid N, and let $P \subseteq N$. Give an algorithm for computing the syntactic monoid of $\varphi^{-1}(P)$.

Exercise 2.3

Let S be a finite semigroup. An element $e \in S$ is *idempotent* if $e^2 = e$.

(a) Show that for every $x \in S$ there exists a unique idempotent element in the set $\{x^k \mid k \geq 1\} \subseteq S$.

(b) Show that $x^{|S|!}$ is idempotent for every $x \in S$.

Exercise 2.4

Let $\varphi : \Sigma^* \to M$ be a homomorphism to a finite monoid M. Show that there exists an integer $n \geq 1$ such that $\varphi(\Sigma^n) = \varphi(\Sigma^{2n})$. As usual, $\varphi(L)$ denotes the subset $\{\varphi(u) \mid u \in L\}$ of M.

Exercise 2.5

Given a word $w \in \Sigma^*$ and a subset $\Gamma \subseteq \Sigma$, we define informally $\pi_\Gamma(w)$ as the word obtained by erasing all letters of w that are not in Γ. More precisely, $\pi_\Gamma : \Sigma^* \to \Gamma^*$ is defined by $\pi_\Gamma(a) = a$ if $a \in \Gamma$ and $\pi_\Gamma(a) = \varepsilon$ otherwise. Show that if $L \subseteq \Sigma^*$ is recognizable, then $\pi_\Gamma(L) = \{\pi_\Gamma(u) \mid u \in L\}$ is recognizable.