Technische Universität München **Winter term 2013/2014**
I7
M. Kufleitner / A. Durand-Gasselin **24.10.2013**

Automata and Formal Languages – Exercise sheet 1

Exercise 1.1

(a) Give all homomorphisms $\varphi : \mathbb{N} \to \Sigma^*$.

(a) Give all homomorphisms $\varphi : \mathbb{Z} \to \Sigma^*$.

Exercise 1.2

Let $\varphi : M \to N$ be a homomorphism and let $L \subseteq M$. Show that $L = \varphi^{-1}(\varphi(L))$ if and only if there exists a subset $P \subseteq N$ with $L = \varphi^{-1}(P)$.

Exercise 1.3

Let Σ be a finite alphabet and let $L \subseteq \Sigma^*$. Show that $\mathrm{Synt}(L)$ has a generating set of size at most $|\Sigma|$.

Exercise 1.4

Show that every group is the syntactic monoid of some language.

Exercise 1.5

Let $M = \{1, a_1, \ldots, a_n\}$ with neutral element 1 and the operation $a_i \cdot a_j = a_j$. Show that M is the syntactic monoid of some language if and only if $n \leq 2$.

Exercise 1.6

Suppose $L \subseteq \Sigma^*$ is recognizable. Show that $\sqrt{L} = \{u \in \Sigma^* \mid uu \in L\}$ is recognizable, too.

Exercise 1.7

Give the syntactic monoid of the language $L = (ab)^*$.

Exercise 1.8

Let $\Sigma = \{a, b, c\}$ and let $L = \{u \in \Sigma^* \mid u \text{ has the same number of } a\text{'s and } b\text{'s}\}$. Give the syntactic monoid of L.

Exercise 1.9

Give the syntactic monoid and the minimal automaton of the language $L = \{a, b\}^n a \{a, b\}^*$.