Exercise 1  

\(6 \times 1.5 \ P = 9 \ P\)

**Question:** Consider \(L \subseteq \{a,b\}^*\) the set of words, where \(a\) occurs only at even, or only at odd positions (not necessarily at all even/odd positions). For example, \(aba \in L, babbb \in L, aa \notin L\). Give a corresponding MSO formula and an automaton for \(L\). You may use macros from the lecture notes.

**Answer:**

**Question:** Let \(L \subseteq \Sigma^*\) be a language defined by the MSO sentence \(\forall X \exists x (x \in X \lor x \notin X)\). Write down a regular expression for \(L\).

**Answer:**

**Question:** Write an MSO formula for the language \((aab)^*\). You may use macros from the lecture notes.

**Answer:**

**Question:** Give a Büchi automaton and a \(\omega\)-regular expression for the formula \(FG(p \lor q)\) over the atomic propositions \(\text{AP} = \{p, q\}\). (Recall that the language of the formula is an \(\omega\)-language over the alphabet \(\Sigma = 2^{\text{AP}}\).)

**Answer:**

**Question:** Decide whether \(G(aUb)\) and \(G(a \lor b) \land Fb\) are equivalent and prove your answer.

**Answer:**

**Question:** Let \(A\) be a DFA recognizing a language \(L \subseteq \Sigma^n\) of a fixed length \(n\). What is the language recognized by \(A\) seen as a co-Büchi automaton? (Recall that transition function of a DFA is total.)

**Answer:**
Exercise 2

(a) Give a transducer $T$ over $\{0,1\}$ recognizing the lsbf encodings of the pairs $(v,w) \in \mathbb{N} \times \mathbb{N}$ such that $v < w$, i.e. $L = \{(v,w) \mid \text{lsbf}^{-1}(v) < \text{lsbf}^{-1}(w)\}$

(b) Prove that $T$ recognizes $L$ by induction on the length of the word.

Exercise 3

Recall that $\{a^m b^n \mid m = n\}$ is not regular. Decide whether the following languages are regular or not. If yes, give a corresponding automaton or a regular expression. If no, either show it has infinitely many residuals, or use closure properties as discussed in the exercises.

(a) $L_1 = \{w \in \{a,b\}^* \mid w \text{ contains as many } a \text{'s as } b \text{'s}\}$

(b) $L_2 = \{w \in \{a,b\}^* \mid w \text{ contains as many } ab \text{'s as } ba \text{'s}\}$

(For example, in $w = abaab$ there are two $ab$'s and one $ba$, hence $w \notin L_2$.)

(c) $L_3 = \{a^m b^n \mid m \leq n, m < 1000\}$

Exercise 4

Given a finite automaton $A = (Q, \Sigma, \delta, q_0, F)$ recognizing a language $L \subseteq \Sigma^*$, construct a transducer $T = (Q', \Sigma', \delta', q'_0, F')$ recognizing $\{(a_1 \cdots a_n, b_1 \cdots b_n) \in \Sigma^* \times \Sigma^* \mid a_1 a_2 \cdots a_n \in L \text{ and } a_1 b_1 a_2 b_2 \cdots a_n b_n \in L\}$.

Exercise 5

Consider the following program $P$ with a binary variable $x$ initialised to 0:

```
loop
1: non-deterministically choose
2: either $x \leftarrow 1$
3: or $x \leftarrow 0$
```

(a) Construct a network of automata for $P$ and $x$ and their asynchronous product.

(b) Using the standard algorithm from the lecture decide whether $F \ x = 1$ holds for $P$.

Exercise 6

Let $\text{Inf}(w)$ denote the set of letters that occur infinitely often in the word $w$. Consider the language $L = \{w \in \{a,b,c\}^\omega \mid a \in \text{Inf}(w) \Rightarrow b \notin \text{Inf}(w)\}$.

(a) Construct a deterministic Muller automaton for $L$ with only two states.

(b) Construct an equivalent Rabin automaton.

Exercise 7

Given a language $L \subseteq \Sigma^*$ of finite words, we define the limit $\omega$-language $\overline{L} \subseteq \Sigma^\omega$ as follows: $w \in \overline{L}$ iff infinitely many prefixes of $w$ belong to $L$. For example, if $L = b + (ab)^*$ then $\overline{L} = (ab)^\omega$.

(a) Give an NFA $\mathcal{N}$ such that the limit of its language and the language of $\mathcal{N}$ viewed as a Büchi automaton differ.

(b) Prove that for a DFA $\mathcal{D}$, the limit of its language and the language of $\mathcal{D}$ viewed as a Büchi automaton coincide.