Automata and Formal Languages – Homework 10

Due 14.1.2013.

Exercise 10.1

Recall that finite languages of finite words are regular.

Find a language over $\{a, b\}$ consisting of one infinite word such that there is no Büchi automaton recognizing it.

Exercise 10.2

Construct Büchi automata accepting the following languages over $\Sigma = \{a, b, c\}$.

(a) $L_0 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains ab exactly once $\}$.

(b) $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains ab at least once $\}$.

(c) $L_2 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains ab infinitely often $\}$.

(d) $L_3 = \{\alpha \in \Sigma^\omega \mid$ contains ab only finitely often $\}$.

(e) $L_4 = \{\alpha \in \Sigma^\omega \mid$ if α contains infinitely many a's then α contains infinitely many b's $\}$.

Exercise 10.3

Construct *deterministic* Büchi automata accepting the following languages over $\Sigma = \{a, b, c\}$.

(a) $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains at least one letter $c\}$.

(b) $L_2 = \{\alpha \in \Sigma^\omega \mid$ in α, every a is immediately followed by a $b\}$.

(c) $L_3 = \{\alpha \in \Sigma^\omega \mid$ in α, between two successive a's there are at least two b's $\}$.

Exercise 10.4

For a word w, let $\inf(w)$ denote the set of letters that occur infinitely many times in w.

Construct a Büchi automaton recognizing the language L over alphabet $\{a, b, c\}$ where

(a) $L = \{w \mid \{a, b\} \supseteq \inf(w)\}$

(b) $L = \{w \mid \{a, b\} = \inf(w)\}$

(c) $L = \{w \mid \{a, b\} \subseteq \inf(w)\}$

(d) $L = \{w \mid \{a, b, c\} = \inf(w)\}$

(e) $L = \{w \mid$ if $a \in \inf(w)$ then $\{b, c\} \subseteq \inf(w)\}$

Hint: It may be easier to construct a generalized Büchi automaton first and then transform it into a Büchi automaton.

Give the corresponding ω-regular expressions, too.

Exercise 10.5

- You are given finite words $u, v, x, y \in \Sigma^*$ which represent the ω-words $w := u\, v^\omega$ and $z := x\, y^\omega$.

  Give an algorithm for deciding "$w \stackrel{?}{=} z$?".
- You are given a Büchi automaton $\mathcal{B}$ and two finite words u, v representing the ω-word $w := uv^\omega$.

  Give an algorithm for deciding "$w \stackrel{?}{\in} \mathcal{L}(\mathcal{B})$".

Exercise 10.6

For $L \subseteq \{a, b\}^\omega$ below, find an ω-regular expression of the form $\bigcup_{i=1}^n U_i V_i^\omega$ representing the language, such that each U_i and V_i are regular languages of finite words.

(a) $L = \{w \mid k \text{ is even for each substring } ba^k b \text{ of } w\}$

(b) $L = \{w \mid w \text{ has no occurrence of } bab\}$

Exercise 10.7

Find a transformation from Büchi automata to Rabin automata and back.

Exercise 10.8

The acceptance consition of *Streett automata* is of the form $\{(F_i, I_i) \mid i \in \{1, ..., n\}\}$ and a run ρ is accepting if for all $i \in \{1, ..., n\}$, whenever $\mathrm{Inf}(\rho) \cap F_i \neq \emptyset$ then also $\mathrm{Inf}(\rho) \cap I_i \neq \emptyset$.

What is the relationship between Rabin and Streett automata?