Technische Universität München **Winter term 2012/13**
I7
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Automata and Formal Languages – Homework 8

Due 17.12.2012.

Exercise 8.1

Characterize the languages described by the following formulae and give corresponding automata:

(a) $\exists x \, \mathit{first}(x)$

(b) $\forall x \, \mathit{first}(x)$

(c) $\big(\neg\exists x\exists y(x < y \wedge Q_a(x) \wedge Q_b(y))\big) \;\wedge\; \big(\forall x(Q_b(x) \rightarrow \exists y(x < y \wedge Q_a(y)))\big) \wedge \; \big(\exists x(\neg\exists y x < y \wedge Q_a(x))\big)$

Exercise 8.2

For the following languages over $\{a, b\}$, write down their defining MSO formula, automaton and regular expression.

- The set of words of even length and containing only a's or only b's.
- The set of words, where between each two b's with no other b in between there is a block of an odd number of letters a.
- The set of words with odd length and an odd number of occurrences of a.

Exercise 8.3

Give formulae expressing the following macros:

(a) $\text{Sing}(X)$ meaning that the set X is a singleton,

(b) $X \subseteq Y$ meaning subset inclusion,

(c) $X \subseteq Q_a$ meaning all elements of X are labelled by a, for $a \in \Sigma$,

(d) $X < Y$ that is true for singletons $X = \{x\}, Y = \{y\}$ satisfying $x < y$.

Exercise 8.4

We interpret the monadic second order logic over finite words with the standard interpretation of $<$ as less than relation.

Let $MSO'(S)$ be a modification of the standard monadic second-order logic given by the following syntax. Assume a set of second-order logical variables ranged over by X, Y, Z. Let Σ be an alphabet. An $MSO'(<)$ formula over Σ is defined by the following BNF, where $a \in \Sigma$:

$$\varphi ::= X \subseteq Q_a \mid X < Y \mid \text{Sing}(X) \mid X \subseteq Y \mid \neg\varphi \mid (\varphi \vee \varphi) \mid \exists X \varphi$$

Although we quantify over set variables only, we want this logic to be equally "powerful" as the original $MSO(<)$. As there are no first-order variables, the first-order predicates $<$ will be replaced by the second-order predicates, so new atomic formulas are introduced: $\text{Sing}(X)$ (meaning singleton), $X \subseteq Y$ (meaning subset inclusion), $X \subseteq Q_a$ for every $a \in \Sigma$ (meaning all elements of X are labelled by a), and $X < Y$ (true for singletons $X = \{x\}, Y = \{y\}$ satisfying $x < y$).

(a) Show that $MSO(<)$ and $MSO'(<)$ are equally expressive, i.e., a language is definable in $MSO(<)$ iff it is definable in $MSO'(<)$.

Hint: Express the newly defined predicates in the original MSO and vice versa.

Remark: This logic can be used to create a different (a bit easier) procedure to translate formulae into automata: the problem of incorrect encodings does not arise.

(b) Translate the formula

$$\exists Z \forall x (Q_a(x) \rightarrow \exists y (x < y \wedge y \in Z))$$

into an equivalent one of $MSO'(<)$.