Technische Universität München **Winter term 2012/13**
I7
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Automata and Formal Languages – Homework 4

Due 19.11.2012.

Exercise 4.1

In the lecture, you have seen that we can save on space using the *lazy* DFAs. However, this does not come for free. There is a space vs. running-time trade-off because of extra steps with head not moving in the case with lazy DFAs.

Find a word w and a pattern p such that the run of **eagerDFA**(p) on w takes at most n steps and the run of **lazyDFA**(p) takes at least $2n - 1$ steps.

Hint: a simple pattern of the form a^k is sufficient.

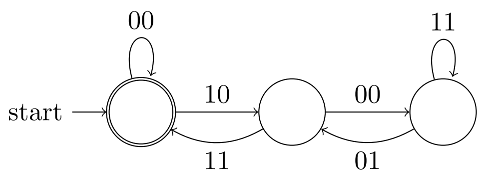
Exercise 4.2

Design an algorithm that solves the following problem for a finite alphabet Σ. Discuss the complexity of your solution.

- *Given*: $w \in \Sigma^*$ and a regular expression r over Σ.
- *Find*: A shortest prefix $w_1 \in \Sigma^*$ of w such that there exists a prefix $w_1 w_2$ of w and $w_2 \in \mathcal{L}(r)$.

Exercise 4.3

Consider the following FA $\mathcal{A}$ over the alphabet $\{00, 01, 10, 11\}$:



W.r.t. the msbf encoding, we may interpret any word $w \in \{00, 01, 10, 11\}^*$ as a pair of natural numbers $(X(w), Y(w)) \in \mathbb{N}_0 \times \mathbb{N}_0$. *Example*: (Underlined letters correspond to $Y(w)$.)

$$w = (00)^k 001011 \to (0\underline{0})^k 0\underline{0}1\underline{0}1\underline{1} \to (0^k 011, 0^k 001) \to (3, 1) = (X(w), Y(w))$$

(a) Show that $w \in \mathcal{L}(\mathcal{A})$ iff $X(w) = 3 \cdot Y(w)$.

(b) Construct the minimal DFA representing the language $\{w \in \{0,1\}^* \mid \text{msbf}^{-1}(w) \text{ is divisible by } 3\}$.