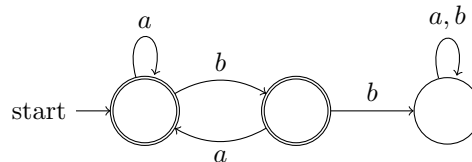


## Automata and Formal Languages – Homework 2

Due 5.11.2012.

### Exercise 2.1

Let  $\mathcal{A}$  be the following finite automaton:



- (a) Transform the automaton  $\mathcal{A}$  into an equivalent regular expression, then transform this expression into an NFA (with  $\varepsilon$ -transitions), remove the  $\varepsilon$ -transitions, and determinize the automaton.
- (b) Use JFLAP to perform the same transformations. Is there any difference?
- (c) Using JFLAP check that your resulting automaton is equivalent to the original one.

### Exercise 2.2

Let  $r$  be the regular expression  $((0 + 1)(0 + 1))^*$  over  $\Sigma = \{0, 1\}$ , where  $+$  denotes choice.

- (a) Describe  $\mathcal{L}(r)$  in words.
- (b) Give a regular expression  $r'$  such that  $\mathcal{L}(r') = \Sigma^* \setminus \mathcal{L}(r)$ .

### Exercise 2.3

For any language  $L$ , let  $L_{\text{pref}}$  and  $L_{\text{suf}}$  denote the languages of all prefixes and all suffixes, respectively, of words in  $L$ . E.g. for  $L = \{abc, d\}$ , we have  $L_{\text{pref}} = \{abc, ab, a, \varepsilon, d\}$  and  $L_{\text{suf}} = \{abc, bc, c, \varepsilon, d\}$ .

- (a) Given an automaton  $\mathcal{A}$ , construct automata  $\mathcal{A}_{\text{pref}}$  and  $\mathcal{A}_{\text{suf}}$  so that  $\mathcal{L}(\mathcal{A}_{\text{pref}}) = \mathcal{L}(\mathcal{A})_{\text{pref}}$  and  $\mathcal{L}(\mathcal{A}_{\text{suf}}) = \mathcal{L}(\mathcal{A})_{\text{suf}}$ .
- (b) Consider a regular expression  $r = (ab + b)^*c$ . Give a regular expression  $r_{\text{pref}}$  so that  $\mathcal{L}(r_{\text{pref}}) = \mathcal{L}(r)_{\text{pref}}$ .

### Exercise 2.4

For  $n \in \mathbb{N}_0$  let  $\text{msbf}(n)$  be the language of all words over  $\{0, 1\}$  which represent  $n$  w.r.t. to the *most significant bit first* representation where an arbitrary number of leading zeros is allowed. For example:

$$\text{msbf}(3) = \mathcal{L}(0^*11) \text{ and } \text{msbf}(0) = \mathcal{L}(0^*).$$

Similarly, let  $\text{lsbf}(n)$  denote the language of all *least significant bit first* representations of  $n$  with an arbitrary number of following zeros, e.g.:

$$\text{lsbf}(6) = \mathcal{L}(0110^*) \text{ and } \text{lsbf}(0) = \mathcal{L}(0^*).$$

- (a) Construct and compare DFAs representing all even natural numbers  $n \in \mathbb{N}_0$  w.r.t. the unary encoding (i.e.,  $n \mapsto a^n$ ), the msbf encoding, and the lsbf encoding.
- (b) Construct a DFA representing the language  $\{w \in \{0, 1\}^* \mid \text{lsbf}^{-1}(w) \text{ is divisible by } 3\}$ .
- (c) Give regular expressions corresponding to the languages in (a) and (b).

### Exercise 2.5

For alphabets  $\Sigma$  and  $\Delta$ , a *substitution* is a mapping  $f : \Sigma \rightarrow 2^{\Delta^*}$  assigning to each letter  $a \in \Sigma$  a language  $L_a \subseteq \Delta^*$ . Then by setting  $f(\varepsilon) = \varepsilon$  and  $f(wa) = f(w)f(a)$  we can define  $f(L) = \bigcup_{w \in L} f(w)$ .

Note that a homomorphism can be identified with a substitution where all  $L_a$ 's are singletons.

Consider the following example. Let  $\Sigma = \{Name, Telephone, :, \#\}$  and  $\Delta = \{A, \dots, Z, 0, 1, \dots, 9, :, \#\}$ . The substitution  $f$  is given by

$$f(Name) = \mathcal{L}((A + \dots + Z)^*)$$

$$f(:) = \{:\}$$

$$f(Telephone) = \mathcal{L}(0049(1 + \dots + 9)(0 + 1 + \dots + 9)^{10} + 00420(1 + \dots + 9)(0 + 1 + \dots + 9)^8)$$

$$f(\#) = \{\#\}$$

- Draw a DFA recognizing  $L = Name : Telephone \{\#Telephone\}^*$ .
- Sketch an NFA-reg recognizing  $f(L)$ .
- Given an automaton recognizing  $L'$ , a substitution  $f'$ , and automata recognizing  $f'(a)$  for every  $a$ , construct an automaton recognizing  $f'(L')$ .

### Exercise 2.6

For  $L_1, L_2$  regular languages over an alphabet  $\Sigma$ , the *left quotient* of  $L_1$  by  $L_2$  is defined by

$$L_2 \setminus L_1 := \{v \in \Sigma^* \mid \exists u \in L_2 : uv \in L_1\}$$

- Use the fact that regular languages are closed under homomorphisms, inverse homomorphisms, concatenation and intersection to prove they are closed under quotienting.
- Given finite automata  $\mathcal{A}_1, \mathcal{A}_2$ , construct an automaton  $\mathcal{A}$  such that

$$\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}_2) \setminus \mathcal{L}(\mathcal{A}_1)$$

- Is there any difference when taking the *right quotient*  $L_1 / L_2 := \{u \in \Sigma^* \mid \exists v \in L_2 : uv \in L_1\}$  ?

### Exercise 2.7

Let  $L_1, L_2$  be regular languages. Determine the inclusion relation between the following languages:

- $L_1$
- $(L_1 / L_2) \cdot L_2$
- $(L_1 \cdot L_2) / L_2$