Automata and Formal Languages – Endterm

Please note: If not stated otherwise, all answers have to be justified.

Exercise 1

10 × 1.5 P = 15 P

**Question:** Decide whether the following holds and write down why.

If $L^*$ is a regular language, then $L$ is regular.

**Answer:**

**Question:** Decide whether the following holds and write down why.

If $R$ is a non-empty regular language and $L \cap R^\omega$ is $\omega$-regular, then $L$ is $\omega$-regular.

**Answer:**

**Question:** Draw a finite automaton for $\{w \in \{a, b, c\}^* \mid$ between every $a$ and a later $b$ there is $c\}$.

**Answer:**

**Question:** Let $L$ be a regular language and $r$ the number of its residuals. Further, let $s$ be the minimal possible number of states of a non-deterministic automaton recognizing $L$. What is the relationship between $r$ and $s$?

**Answer:**
Question: Construct a transducer recognizing the relation \( \{(wa, aw) \mid w \in \{a, b\}^*\} \).

Answer: 

Question: Give a regular expression for the language \( L \subseteq \{a, b\}^* \) of words, in which the number of \( ab \)'s is the same as the number of \( ba \)'s. (Examples: \( aaba \in L \), \( abb \notin L \).)

Answer: 

Question: Write down an \( \omega \)-regular expression equivalent to \( G(a \lor Fb) \) over \( Ap = \{a, b\} \).

Answer: 

Question: Let \( \Sigma = \{a, b\} \) be an alphabet. Is there a language \( L \subseteq \Sigma^\omega \) with exactly two words such that \( \Sigma^\omega \setminus L \) is \( \omega \)-regular?

Answer: 

Question: Construct a DFA recognizing the solution space of \( 2x \leq y \) in the lsbf encoding.

Answer: 

Question: Decide whether the formulae \( G(aUb) \) and \( G(a \lor b) \) are equivalent. Why?

Answer: 
Exercise 2

Let $L \subseteq \Sigma^*$ and $M \subseteq \Gamma^*$ be languages. We define the shuffle of $L$ and $M$ as

$$L || M := \{ u_1v_1 \cdots u_nv_n \mid n \in \mathbb{N}, u_1 \cdots u_n \in L, v_1 \cdots v_n \in M, \forall 1 \leq i \leq n \ u_i \in \Sigma^*, v_i \in \Gamma^* \}$$

Example: Let $\Sigma = \{a, b\}$, $L = \{ab\}$ and $\Gamma = \{0, 1\}$, $M = \{0, 1\}$. Then $L || M = \{ab0, a0b, 0ab, ab1, a1b, 1ab\}$.

Prove that the shuffle operation preserves regularity, i.e. if $L$ and $M$ are regular then so is $L || M$.

Exercise 3

Give regular expressions for the complements of the following languages over $\Sigma = \{a, b\}$ and explain your answers:

(a) $(aa + bb)^*$
(b) $(a + b)^*(aa + bb)(a + b)^*$

Exercise 4

(a) Let $\Sigma = \{a, b\}$. Write down all residuals of $(aa^* + \varepsilon)b\Sigma^*$.
(b) Let $\Sigma = \{a\}$. Prove that $\{a^{2n} \mid n \in \mathbb{N}\}$ has infinitely many residuals.

Exercise 5

Consider the following program $P$ with a binary variable $x$ initialised to 0:

P:

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loop
1: x ← 1
2: x ← 0
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(a) Construct a network of automata for $P$ and $x$ and their asynchronous product.
(b) Construct a Büchi automaton for the negation of the property $G F x = 0$.
(c) Using the product construction, prove that the previous property holds for $P$.

Exercise 6

Order the following languages of infinite words over $\Sigma = \{a, b, c, d\}$ with respect to the set inclusion $\subseteq$.

- $L_1 = \{ w \in \Sigma^* \mid \inf(w) = \{b, c\} \}$
- $L_2 = \{ w \in \Sigma^* \mid \{b, c\} \subseteq \inf(w) \}$
- $L_3 = \{ w \in \Sigma^* \mid \inf(w) \subseteq \{a, b, c, d\} \}$
- $L_4 = \{ w \in \Sigma^* \mid d \notin \inf(w) \land \{b, c\} \subseteq \inf(w) \}$

Further, construct an NGA recognizing one of those (and indicate which one).
Exercise 7

Recall that NestedDFS and TwoStack are non-deterministic algorithms that stop after reporting the first accepting lasso found. Consider the following automaton:

(a) Which lassos can be reported by the NestedDFS algorithm? Why?
(b) Which lassos can be reported by the TwoStack algorithm? Why

Exercise 8

Consider the language $L$ of the regular expression $(ab)^*$. 
(a) Give an MSO formula for $L$. 
(b) Give an MSO formula for $L$ with no second-order variables.