Technische Universität München **Winter term 2011/12**
I7
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Automata and Formal Languages – Homework 11

Due 19.1.2012.

Exercise 11.1

Implement the intersection of Büchi automata with a simple procedure using *NGAtoNBA*. Further, can you make use of this procedure when intersecting more than two automata?

Exercise 11.2

For this exercise, let $\Sigma := \{a, b\}$. Consider the ω-regular expression

$$\phi_k := ((\Sigma^{k+1})^* \Sigma^k a)^\omega \text{ with } k \geq 1.$$

(a) Describe $\mathcal{L}(\phi_k)$ in words.

(b) Construct a Büchi automaton $\mathcal{B}_k$ s.t. $\mathcal{L}(\mathcal{B}_k) = \mathcal{L}(\phi_k)$.

(c) Apply the intersection construction to $\mathcal{B}_1$ and $\mathcal{B}_2$.

(d) Can you come up with a Büchi automaton for $\mathcal{L}(\phi_1) \cap \mathcal{L}(\phi_2)$ which has less states then the one obtained in (c)?

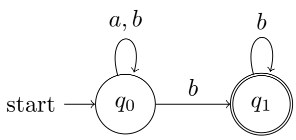
Exercise 11.3

Construct a generalized Büchi and a Büchi automaton accepting $L_1 \cap L_2 \cap L_3 \subseteq \{a, b\}^\omega$, where

- $L_1 = \{\alpha \mid \text{infinitely many } a\text{'s occur in } \alpha\}$
- $L_2 = \{\alpha \mid \text{finitely many } b\text{'s occur in } \alpha\}$
- $L_3 = \{\alpha \mid \text{each } a \text{ in } \alpha \text{ is immediately followed by a } b\}$

Exercise 11.4

Consider the following Büchi $\mathcal{B}$ automaton representing the ω-words over $\Sigma = \{a, b\}$ having only finitely many as:



(a) Sketch $\text{dag}(abab^\omega)$ and $\text{dag}((ab)^\omega)$.

(b) Consider the ranking r defined by $r(\langle q_0, i\rangle) := 1$ and $r(\langle q_1, i\rangle) := 0$ for all $i \in \mathbb{N}$.

Is r an odd ranking for $\text{dag}(abab^\omega)$, resp. $\text{dag}((ab)^\omega)$?

(c) Show that the ranking r defined in (b) is odd for $\text{dag}(w)$ iff $w \notin \mathcal{L}(\mathcal{B})$.

(d) Apply now the complement construction for Büchi automata to $\mathcal{B}$ as seen in the lecture. Hint: You may use the fact that it is sufficient to use $\{0, 1\}$ as ranks.