

Automata and Formal Languages – Homework 10

Due 12.1.2012.

Exercise 10.1

Recall that finite languages of finite words are regular.

Find a language over $\{a, b\}$ consisting of one infinite word such that there is no Büchi automaton recognizing it.

Exercise 10.2

Construct Büchi automata accepting the following languages over $\Sigma = \{a, b, c\}$.

- (a) $L_0 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains } ab \text{ exactly once}\}$.
- (b) $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains } ab \text{ at least once}\}$.
- (c) $L_2 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains } ab \text{ infinitely often}\}$.
- (d) $L_3 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains } ab \text{ only finitely often}\}$.
- (e) $L_4 = \{\alpha \in \Sigma^\omega \mid \text{if } \alpha \text{ contains infinitely many } a\text{'s then } \alpha \text{ contains infinitely many } b\text{'s}\}$.

Exercise 10.3

Construct *deterministic* Büchi automata accepting the following languages over $\Sigma = \{a, b, c\}$.

- (a) $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains at least one letter } c\}$.
- (b) $L_2 = \{\alpha \in \Sigma^\omega \mid \text{in } \alpha, \text{ every } a \text{ is immediately followed by a } b\}$.
- (c) $L_3 = \{\alpha \in \Sigma^\omega \mid \text{in } \alpha, \text{ between two successive } a\text{'s there are at least two } b\text{'s}\}$.

Exercise 10.4

For a word w , let $\text{inf}(w)$ denote the set of letters that occur infinitely many times in w .

Construct a Büchi automaton recognizing the language L over alphabet $\{a, b, c\}$ where

- (a) $L = \{w \mid \{a, b\} \supseteq \text{inf}(w)\}$
- (b) $L = \{w \mid \{a, b\} = \text{inf}(w)\}$
- (c) $L = \{w \mid \{a, b\} \subseteq \text{inf}(w)\}$
- (d) $L = \{w \mid \{a, b, c\} = \text{inf}(w)\}$
- (e) $L = \{w \mid \text{if } a \in \text{inf}(w) \text{ then } \{b, c\} \subseteq \text{inf}(w)\}$

Hint: It may be easier to construct a generalized Büchi automaton first and then transform it into a Büchi automaton.

Give the corresponding ω -regular expressions, too.

Exercise 10.5

- You are given finite words $u, v, x, y \in \Sigma^*$ which represent the ω -words $w := uv^\omega$ and $z := xy^\omega$.
Give an algorithm for deciding “ $w \stackrel{?}{=} z$?”.
- You are given a Büchi automaton \mathcal{B} and two finite words u, v representing the ω -word $w := uv^\omega$.
Give an algorithm for deciding “ $w \stackrel{?}{\in} \mathcal{L}(\mathcal{B})$ ”.

Exercise 10.6

For $L \subseteq \{a, b\}^\omega$ below, find an ω -regular expression of the form $\bigcup_{i=1}^n U_i V_i^\omega$ representing the language, such that each U_i and V_i are regular languages of finite words.

- (a) $L = \{w \mid k \text{ is even for each substring } ba^k b \text{ of } w\}$
- (b) $L = \{w \mid w \text{ has no occurrence of } bab\}$