Technische Universität München **Winter term 2011/12**
I7
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Automata and Formal Languages – Homework 7

Due 1.12.2010.

Exercise 7.1

A DFA is *synchronizing* if there is a word w and a state q such that after reading w from any state, we are always in state q.

(a) Give an algorithm to decide if a given DFA is synchronizing.

(b) Give a *polynomial* algorithm to decide if a given DFA is synchronizing.

Exercise 7.2

Let $\Sigma = \{request, answer, working, idle\}$.

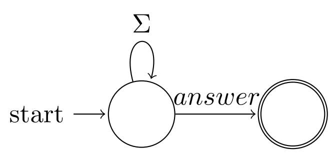
(a) Build an automaton recognizing all words with the property P_1: after every *request* there is *answer* later on (not necessarily immediately).

Does it guarantee that every *request* has its own *answer*? More precisely, let us denote $w = w_1 w_2 \cdots w_n$ and assume that there are k *request*s. Let us define $f : \{1, \dots, k\} \to \{1, \dots, m\}$ such that $w_{f(i)}$ is the ith *request* in w. Provided w satisfies P_1, is there always an injective function $g : \{1, \dots, k\} \to \{1, \dots, m\}$ satisfying $w_{g(i)} = answer$ and $f(i) < g(i)$ for all $i \in \{1, \dots, k\}$?

If words were infinite and there were infinitely many *request*s, would P_1 guarantee that every *request* has its own *answer*? More precisely, let us denote $w = w_1 w_2 \cdots$ and assume that there are infinitely many *request*s. Let us define $f : \mathbb{N} \to \mathbb{N}$ such that $w_{f(i)}$ is the ith *request* in w. Provided w satisfies P_1, is there always an injective function $g : \mathbb{N} \to \mathbb{N}$ satisfying $w_{g(i)} = answer$ and $f(i) < g(i)$ for all $i \in \{1, \dots, k\}$?

(b) Build an automaton recognizing all words with the property P_2: there is an *answer* and before that there are only *working*s and *request*s.

(c) Let $\mathcal{A}$ be the following automaton



Using the intersection construction, prove that all accepting runs of $\mathcal{A}$ satisfy P_1 and find all accepting runs violating P_2.

Exercise 7.3

This exercise focuses on modelling and verification of mutual exclusion protocols. Let us consider having two agents, one having his internal variable id set to 0, the other has her variable id set to 1. They both run the following mutex program:

```
while(true)
    enter(id)
    critical-command
    leave(id)
    loop-arbitrarily-many-times
        non-critical-command
```

The definitions of procedures enter(int) and leave(int) as well as global variables used and their initial values are specified below.

(a)
```
int turn:=0
proc enter(int i){
    while(turn=1-i) do
        skip
}
proc leave(int i){
    turn:=1-i
}
```

Design an asynchronous network of automata capturing this behaviour.

Furthermore, build an automaton recognizing all runs reaching a configuration with both agents in the critical section. Using the intersection algorithm, prove that there are no such runs of this system, i.e. it is a *mutex* algorithm.

Do all infinite runs satisfy that if a process wants to enter the critical section then it eventually enters it?

(b)
```
bool flag[0]:=false
bool flag[1]:=false
proc enter(int i){
    flag[i]:=true
    while(flag[1-i]) do
        skip
}
proc leave(int i){
    flag[i]:=false
}
```

Design an asynchronous network of automata capturing this behaviour.

Can a deadlock occur?

(c) Peterson's algorithm combines both approaches:

```
int turn:=0
bool flag[0]:=false
bool flag[1]:=false
proc enter(int i){
    turn:=1-i
    flag[i]:=true
    while(flag[1-i] & turn=1-i)
        skip
}
proc leave(int i){
    flag[i]:=false
}
```

Can a deadlock occur?

What kind of starving can occur?