Automata and Formal Languages – Endterm

Please note: If not stated otherwise, all answers have to be justified.

Exercise 1

Each of the following questions admits an answer fitting in one or two lines.

(a) Prove or disprove: “Every regular language is recognized by an NFA with all states final.”

(b) For every $n \in \mathbb{N}$, let us define a relation $R_n = \{(u, v) \mid \text{lsbf}^{-1}(u) = n \cdot \text{lsbf}^{-1}(v)\}$.

Assuming $R_2$ and $R_3$ are regular, prove that $R_6$ is regular.

(c) Let $\Sigma = \{a, b\}$ be an alphabet. Give an MSO formula defining the language of $\Sigma(a\Sigma \Sigma)^*a$.

You may use any macros defined in the “Skript”.

(d) Construct a Büchi automaton recognizing the following language:

$$L = \{w \in \{a, b, c, d\}^* \mid a, b \in \text{inf}(w) \text{ and } d \notin \text{inf}(w)\}$$

where $\text{inf}(w)$ denotes the set of letters that appear infinitely many times in $w$.

(e) Describe a procedure to complement deterministic Muller automata directly without using any translation to Büchi automata.

(f) Consider the following NBA $A$.

![Diagram of NBA A]

The following sequences are accepting lassos of $A$. The cycles are underlined.

i) 010

ii) 0210

iii) 02101

iv) 0233

Which of the lassos can be found by a run of NestedDFS on $A$?

Exercise 2

Construct an eagerDFA (not a lazyDFA) for the word mamamia over the alphabet $\{m, a, i\}$. 

Exercise 3

Consider a variable $x$ with domain $\{0, 1, 2\}$ initialized to 0 and the following program with two parallel processes:

<table>
<thead>
<tr>
<th>Process 1:</th>
<th>Process 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>loop</td>
<td>loop</td>
</tr>
<tr>
<td>1: $x \leftarrow 1$</td>
<td>1: $x \leftarrow 2$</td>
</tr>
<tr>
<td>2: $x \leftarrow 0$</td>
<td></td>
</tr>
</tbody>
</table>

(a) Construct the corresponding network of the three automata and their asynchronous product.

(b) Consider a set of atomic propositions $AP = \{x = 0, x = 1, x = 2\}$. Construct a Büchi automaton over $AP$ corresponding to the property that from some point on $x = 0$ holds forever. Give the corresponding LTL formula, too.

(c) Is there an $\omega$-execution of the program that satisfies the property in (b)? Why?/Why not?

Exercise 4

Consider languages over $\{0, 1\}$ with fixed length of 3.

(a) Construct a fragment of the master automaton for the language $L \subseteq \{0, 1\}^3$ of msbf binary encodings of all prime numbers in the range from 0 to 7. (Recall that the smallest prime number is 2.)

(b) Is there a language $L \subseteq \{0, 1\}^3$ with a minimal DFA having 10 states?

Exercise 5

Prove that the following finite automaton over $\{0, 1, 2\}$ accepts precisely the msbf ternary encodings of even numbers. (E.g. 211 is accepted because $2 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0 = 22$ is even.) Proceed by induction on the length of the word.

```
  0, 2
start --q-- 1 --> r
  0, 2
```

Exercise 6

A DFA is synchronizing if there is a word $w$ and a state $q$ such that after reading $w$ from any state we are always in state $q$.

(a) Give such a word $w$ showing that the following DFA is synchronizing.

```
  b
start --a-- a --> a
a --b-- b

  b
```

(b) Give an algorithm to decide if a given DFA is synchronizing.

(c) Give a polynomial time algorithm to decide if a given DFA is synchronizing.