Technische Universität München | **Winter term 2010/11**
I7
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Automata and Formal Languages – Homework 12

Due 3.2.2011.

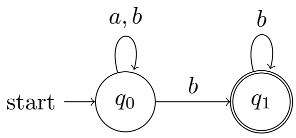
Exercise 12.1

When interpreting MSO over finite words, we of course quantify the second-order variables over *finite* sets only. When applying it to infite words, we can either keep this kind of quantification which is sometimes called *weak monadic second-order logic*, or we can quantify over arbitrary (also infinite) sets which is the usual interpretation of MSO.

Give an example of a formula that is a tautology in $MSO(\prec)$ interpreted over the structure $(\mathbb{N}, <)$ and a contradiction in $WMSO(\prec)$.

Exercise 12.2 **

Consider the following Büchi $\mathcal{B}$ automaton representing the ω-words over $\Sigma = \{a, b\}$ having only finitely many as:



(a) Sketch $\text{dag}(abab^\omega)$ and $\text{dag}((ab)^\omega)$.

(b) Consider the ranking r defined by $r(\langle q_0, i\rangle) := 1$ and $r(\langle q_1, i\rangle) := 0$ for all $i \in \mathbb{N}$.

Is r an odd ranking for $\text{dag}(abab^\omega)$, resp. $\text{dag}((ab)^\omega)$?

(c) Show that the ranking r defined in (b) is odd for $\text{dag}(w)$ iff $w \notin \mathcal{L}(\mathcal{B})$.

(d) Apply now the complement construction for Büchi automata to $\mathcal{B}$ as seen in the lecture. Hint: You may use the fact that it is sufficient to use $\{0, 1\}$ as ranks.

Exercise 12.3 *

Run the emptiness algorithms from the lecture on

- automata from 11.2,
- $(\{0, 1, 2, 3, 4\}, \delta, 0, \{2\})$, where $\delta(0) = \{4\}, \delta(1) = \{3\}, \delta(2) = \{1\}, \delta(3) = \{1\}, \delta(4) = \{0, 2\}$.