Technische Universität München **Winter term 2010/11**
I7
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Automata and Formal Languages – Homework 11

Due 27.1.2011.

For a word w, let $inf(w)$ denote the set of letters that occur infinitely many times in w.

Exercise 11.1

Recall that finite languages of finite words are regular.

Find a language over $\{a, b\}$ consisting of one infinite word such that there is no Büchi automaton recognizing it.

Exercise 11.2

Construct a Büchi automaton recognizing the language L over alphabet $\{a, b, c\}$ where

(a) $L = \{w \mid \{a, b\} \subseteq inf(w)\}$

(b) $L = \{w \mid \{a, b\} = inf(w)\}$

(c) $L = \{w \mid \{a, b\} \supseteq inf(w)\}$

(d) $L = \{w \mid \{a, b, c\} = inf(w)\}$

(e) $L = \{w \mid \text{if } a \in inf(w) \text{ then } \{b, c\} \subseteq inf(w)\}$

Hint: It may be easier to construct a generalized Büchi automaton first and then transform it into a Büchi automaton.

Give the corresponding ω-regular expressions, too.

Exercise 11.3

- You are given finite words $u, v, x, y \in \Sigma^*$ which represent the ω-words $w := u\,v^\omega$ and $z := x\,y^\omega$.

  Give an algorithm for deciding "$w \stackrel{?}{=} z$?".
- You are given a Büchi automaton $\mathcal{B}$ and two finite words u, v representing the ω-word $w := uv^\omega$.

  Give an algorithm for deciding "$w \stackrel{?}{\in} \mathcal{L}(\mathcal{B})$".

Exercise 11.4

For $L \subseteq \{a, b\}^\omega$ below, find an ω-regular expression of the form $\bigcup_{i=1}^{n} U_i V_i^\omega$ representing the language, such that each U_i and V_i are regular languages of finite words.

(a) $L = \{w \mid k \text{ is even for each substring } ba^k b \text{ of } w\}$

(b) $L = \{w \mid w \text{ has no occurrence of } bab\}$