Technische Universität München **Winter term 2010/11**
I7
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Automata and Formal Languages – Homework 7

Due 9.12.2010.

Exercise 7.1

Let $\Sigma = \{0, 1\}$ and for $a, b \in \Sigma$ we define $a \cdot b$ to be the usual multiplication (also an analog of Boolean *and*) and $a \oplus b$ to be 0 if $a = b = 0$, and 1 otherwise (an analog of Boolean *or*).

Consider the following function $f : \Sigma^6 \to \Sigma$ defined by

$$f(x_1, x_2, x_3, x_4, x_5, x_6) = (x_1 \cdot x_2) \oplus (x_3 \cdot x_4) \oplus (x_5 \cdot x_6)$$

(a) Construct the minimal DFA recognizing $L_1 = \{x_1x_2x_3x_4x_5x_6 \mid f(x_1, x_2, x_3, x_4, x_5, x_6) = 1\}$.

(b) Construct the minimal DFA recognizing $L_2 = \{x_1x_3x_5x_2x_4x_6 \mid f(x_1, x_2, x_3, x_4, x_5, x_6) = 1\}$.

Note the difference in the ordering! Give an example of $w \in L_1 \setminus L_2$.

Note the diffrence in the size of the automata. More generally, consider

$$f(x_1, \ldots, x_{2n}) = \bigoplus_{1 \le k \le n} (x_{2k-1} \cdot x_{2k})$$

and languages according to orderings $x_1x_2 \ldots x_{2n-1}x_{2n}$ and $x_1x_3 \ldots x_{2n-1}x_2x_4 \ldots x_{2n}$. Although both languages encode "equivalent" information, their minimal automata differ vastly in the size: for the former ordering the size is linear in n, wheareas for the latter it is exponential.

Exercise 7.2

(a) Given two minimal DFAs accepting bounded languages L_1 and L_2 with words of length k, construct a minimal DFA accepting $L_1 \cup L_2$.

(b) For any language $L \subseteq \{0, 1\}^k$ of binary numbers of length k, we define $L+1$ to be the language $\{w+1 \mod 2^k \mid w \in L\}$. Construct a minimal DFA accepting $L + 1$ from a minimal DFA accepting L.

(c) Let $A = (Q, \{0, 1\}, \delta, q_0, F)$ be a minimal DFA such that $\mathcal{L}(A)$ is a bounded language of binary numbers. What language is accepted by the automaton $A' = (Q, \{0, 1\}, \delta', q_0, F)$, where $\delta'(q, b) = \delta(q, 1 - b)$?