Technische Universität München **Winter term 2010/11**
I7
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Automata and Formal Languages – Homework 3

Due 11.11.2010.

Exercise 3.1

Let $L_i = \{w \in \{a\}^* \mid \text{ the length of } w \text{ is divisible by } i\ \}$.

(a) Construct an NFA for $L := L_4 \cup L_6$ with at most 11 states.

(b) Construct the minimal DFA for L.

Exercise 3.2

Let us consider $\Sigma = \{0, 1\}$ and the msbf encoding.

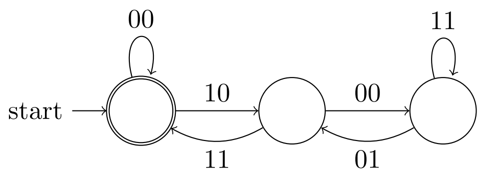
(a) Construct the minimal DFAs accepting the languages L_1, L_2, and L_3^2 defined below.

- $L_1 = \{w \mid \text{msbf}^{-1}w \mod 3 = 0\} \cap \Sigma^4$.
- $L_2 = \{w \mid \text{msbf}^{-1}w \text{ is a prime }\} \cap \Sigma^4$.
- $L_3^k = \{ww \mid w \in \Sigma^k\}$.

(b) How many states has the minimal DFA accepting L_3^k with respect to k?.

Exercise 3.3

Consider the following FA $\mathcal{A}$ over the alphabet $\{00, 01, 10, 11\}$:



W.r.t. the msbf encoding, we may interpret any word $w \in \{00, 01, 10, 11\}^*$ as a pair of natural numbers $(X(w), Y(w)) \in \mathbb{N}_0 \times \mathbb{N}_0$. *Example*: (Underlined letters correspond to $Y(w)$.)

$$w = (00)^k001011 \rightarrow (0\underline{0})^k0\underline{0}1\underline{0}1\underline{1} \rightarrow (0^k011, 0^k001) \rightarrow (3, 1) = (X(w), Y(w))$$

(a) Show that $w \in \mathcal{L}(\mathcal{A})$ iff $X(w) = 3 \cdot Y(w)$.

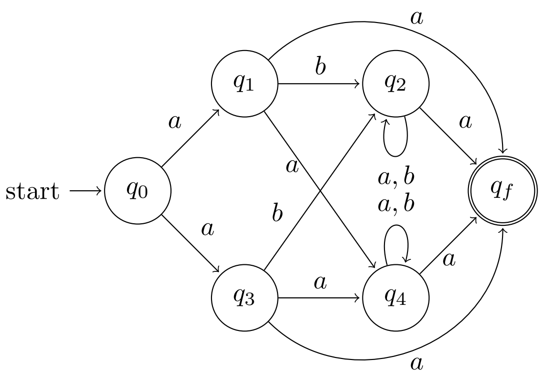
(b) Construct the minimal DFA representing the language $\{w \in \{0, 1\}^* \mid \text{msbf}^{-1}(w) \text{ is divisible by } 3\}$.

Exercise 3.4

Consider the partitioning algorithm from the lecture. Its while-loop clearly cannot be executed more than $|Q| - 1$ times. Show that this bound is tight, i.e. give an example where it is executed $|Q| - 1$ times. (Hint: It is sufficient to consider one-letter alphabet.)

Exercise 3.5

Consider the following NFA $\mathcal{A}$:



(a) Describe $\mathcal{L}(\mathcal{A})$.

(b) Determine the CSR of $\mathcal{A}$ using the algorithm presented in the lecture.