Technische Universität München **Winter term 2010/11**
I7
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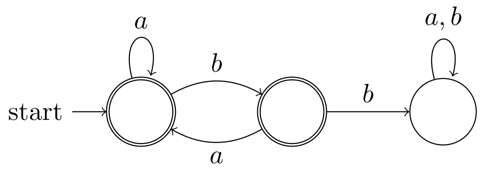
Automata and Formal Languages – Homework 2

Due 4.11.2010.

Exercise 2.1

Let $\mathcal{A}$ be the following finite automaton:



(a) Transform the automaton $\mathcal{A}$ into an equivalent regular expression, then transform this expression into an NFA (with ε-transitions), remove the ε-transitions, and determinize the automaton.

(b) Use JFLAP to perform the same transformations. Is there any difference?

(c) Using JFLAP check that your resulting automaton is equivalent to the original one.

Exercise 2.2

Let r be the regular expression $((0+1)(0+1))^*$ over $\Sigma = \{0,1\}$, where $+$ denotes choice.

(a) Describe $\mathcal{L}(r)$ in words.

(b) Give a regular expression r' such that $\mathcal{L}(r') = \Sigma^* \setminus \mathcal{L}(r)$.

Exercise 2.3

For any language L, let L_{pref} and L_{suf} denote the languages of all prefixes and all suffixes, respectively, of words in L. E.g. for $L = \{abc, d\}$, we have $L_{\text{pref}} = \{abc, ab, a, \varepsilon, d\}$ and $L_{\text{suf}} = \{abc, bc, c, \varepsilon, d\}$.

(a) Given an automaton $\mathcal{A}$, construct automata $\mathcal{A}_{\text{pref}}$ and $\mathcal{A}_{\text{suf}}$ so that $\mathcal{L}(\mathcal{A}_{\text{pref}}) = \mathcal{L}(\mathcal{A})_{\text{pref}}$ and $\mathcal{L}(\mathcal{A}_{\text{suf}}) = \mathcal{L}(\mathcal{A})_{\text{suf}}$.

(b) Consider a regular expression $r = (ab+b)^*c$. Give a regular expression r_{pref} so that $\mathcal{L}(r_{\text{pref}}) = \mathcal{L}(r)_{\text{pref}}$.

Exercise 2.4

For $n \in \mathbb{N}_0$ let $\text{msbf}(n)$ be the language of all words over $\{0,1\}$ which represent n w.r.t. to the *most significant bit first* representation where an arbitrary number of leading zeros is allowed. For example:

$$\text{msbf}(3) = \mathcal{L}(0^*11) \text{ and } \text{msbf}(0) = \mathcal{L}(0^*).$$

Similarly, let $\text{lsbf}(n)$ denote the language of all *least significant bit first* representations of n with an arbitrary number of following zeros, e.g.:

$$\text{lsbf}(6) = \mathcal{L}(0110^*) \text{ and } \text{lsbf}(0) = \mathcal{L}(0^*).$$

(a) Construct and compare DFAs representing all even natural numbers $n \in \mathbb{N}_0$ w.r.t. the unary encoding (i.e., $n \mapsto a^n$), the msbf encoding, and the lsbf encoding.

(b) Construct a DFA representing the language $\{w \in \{0,1\}^* \mid \text{lsbf}^{-1}(w) \text{ is divisible by } 3\}$.

(c) Give regular expressions corresponding to the languages in (a) and (b).

Exercise 2.5

For alphabets Σ and Δ, a *substitution* is a mapping $f : \Sigma \to 2^{\Delta^*}$ assigning to each letter $a \in \Sigma$ a language $L_a \subseteq \Delta^*$. Then by setting $f(\varepsilon) = \varepsilon$ and $f(wa) = f(w)f(a)$ we can define $f(L) = \bigcup_{w \in L} f(w)$.

Note that a homomorphism can be identified with a subsitution where all L_a's are singletons.

Consider the following example. Let $\Sigma = \{Name, Telephone, :, \#\}$ and $\Delta = \{A, \ldots, Z, 0, 1, \ldots, 9, :, \#\}$. The substitution f is given by

$$\begin{aligned}
&f(Name) = \mathcal{L}((A + \cdots + Z)^*)\\
&f(:) = \{:\}\\
&f(Telephone) = \mathcal{L}(0049(1 + \ldots + 9)(0 + 1 + \ldots + 9)^{10} + 00420(1 + \ldots + 9)(0 + 1 + \ldots + 9)^{8})\\
&f(\#) = \{\#\}
\end{aligned}$$

(a) Draw a DFA recognizing $L = Name : Telephone\{\#Telephone\}^*$.

(b) Sketch an NFA-reg recognizing $f(L)$.

(c) Given an automaton recognizing L', a substitution f', and automata recognizing $f'(a)$ for every a, construct an automaton recognizing $f'(L')$.