Technische Universität München **Winter term 2009/10**
I7
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Automata and Formal Languages – Homework 12

Due 4.2.2010.

Exercise 12.1

Recall that finite languages of finite words are regular.

Find a language over $\{a, b\}$ consisting of one infinite word such that there is no Büchi automaton recognizing it.

Exercise 12.2

Construct a Büchi automaton recognizing the language $L = \{w \in \{a, b, c\}^\omega \mid \text{if } a \in inf(w) \text{ then } \{b, c\} \subseteq inf(w)\}$.

Exercise 12.3

For this exercise, let $\Sigma := \{a, b\}$. Consider the ω-regular expression

$$\phi_k := ((\Sigma^{k+1})^* \Sigma^k a)^\omega \text{ with } k \geq 1.$$

(a) Describe $\mathcal{L}(\phi_k)$ in words.

(b) Construct a Büchi automaton $\mathcal{B}_k$ s.t. $\mathcal{L}(\mathcal{B}_k) = \mathcal{L}(\phi_k)$.

(c) Apply the intersection construction to $\mathcal{B}_1$ and $\mathcal{B}_2$.

(d) Can you come up with a Büchi automaton for $\mathcal{L}(\phi_1) \cap \mathcal{L}(\phi_2)$ which has less states then the one obtained in (c)?

Exercise 12.4

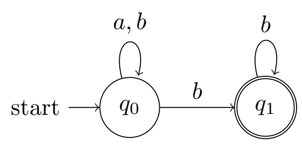
- You are given finite words $u, v, x, y \in \Sigma^*$ which represent the ω-words $w := u\,v^\omega$ and $z := x\,y^\omega$.

  Give an algorithm for deciding "$w \stackrel{?}{=} z$?".
- You are given a Büchi automaton $\mathcal{B}$ and two finite words u, v representing the ω-word $w := uv^\omega$.

  Give an algorithm for deciding "$w \stackrel{?}{\in} \mathcal{L}(\mathcal{B})$".

Exercise 12.5

Consider the following Büchi $\mathcal{B}$ automaton representing the ω-words over $\Sigma = \{a, b\}$ having only finitely many as:



(a) Sketch dag($abab^\omega$) and dag($(ab)^\omega$).

(b) Consider the ranking r defined by $r(\langle q_0, i\rangle) := 1$ and $r(\langle q_1, i\rangle) := 0$ for all $i \in \mathbb{N}$.

Is r an odd ranking for dag($abab^\omega$), resp. dag($(ab)^\omega$)?

(c) Show that the ranking r defined in (b) is odd for dag(w) iff $w \notin \mathcal{L}(\mathcal{B})$.

(d) Apply now the complement construction for Büchi automata to $\mathcal{B}$ as seen in the lecture but use the fact that it is sufficient to use $\{0, 1\}$ as ranks.