Automata and Formal Languages – Homework 6

Due 3.12.2009.

Exercise 6.1

With transducers defined to be finite automata whose transitions are labeled by pairs of symbols $(a, b) \in \Sigma \times \Sigma$ only pairs of words $(a_0a_1 \dots a_l, b_0b_1 \dots b_l)$ of same length can be accepted. Consider therefore finite automata whose transitions are labeled by elements of $(\Sigma \cup \{\varepsilon\}) \times (\Sigma \cup \{\varepsilon\})$ instead, and call this class ε-transducers. As in the case of transducers, we say that an ε-transducer $\mathcal{A}$ accepts a word pair (w, w') if there is a run

$$q_0 \xrightarrow{(a_0,b_0)} q_1 \xrightarrow{(a_1,b_1)} \dots \xrightarrow{(a_n,b_n)} q_n \text{ with } a_i, b_i \in \Sigma \cup \{\varepsilon\}$$

such that $w = a_0a_1 \dots a_n$ and $w' = b_0b_1 \dots b_n$. Note that $|w| \le n$ and $|w'| \le n$. As usual, we write $\mathcal{L}(\mathcal{A})$ for the language of word pairs accepted by the ε-transducer $\mathcal{A}$.

(a) Construct ε-transducer $\mathcal{A}_1$ and $\mathcal{A}_2$ such that $\mathcal{L}(\mathcal{A}_1) = \{(a^nb^m, c^{2n}) \mid n, m \ge 0\}$, and $\mathcal{L}(\mathcal{A}_2) = \{(a^nb^m, c^{2m}) \mid n, m \ge 0\}$.

(b) Apply the construction for the intersection of two finite automata to $\mathcal{A}_1$ and $\mathcal{A}_2$. Which language does the resulting ε-transducer accept?

(c) Show that there is no ε-transducer which accepts the language $\mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\mathcal{A}_2)$.

Exercise 6.2

As seen in the lecture, when applying the post, pre or join operations to transducers the underlying projection operation might yield an automaton which does not accept all possible encodings anymore. You have seen how to fix this probelm in the case that the representations are obtained by padding on the right, as for instance in the lsbf-representation of natural numbers where all representations of a number $n \in \mathbb{N}$ are obtained by adding 0s. In the case of the msbf-encoding, the padding does not occur on the right, but on the left. Hence, the procedure given in the lecture cannot be applied anymore.

- Give an algorithm for calculating the "pad-closure" of a transducer when using the msbf-encoding of natural numbers.