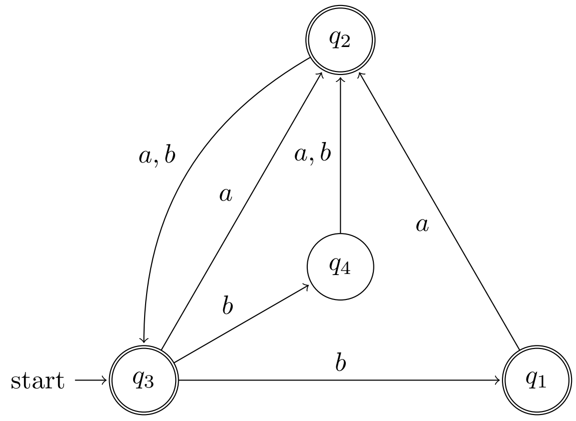
Automata and Formal Languages – Homework 5

Due 26.11.2009.

Exercise 5.1

Check whether the NFA depicted below recognizes Σ^* by means of the algorithm "UnivNFA" presented in the lecture.



Exercise 5.2

We define the following languages over the alphabet $\Sigma = \{a, b\}$:

- L_1 is the set of all words where between any two occurrences of b's there is at least one a.
- L_2 is the set of all words where every maximal sequence of consecutive a's has odd length.
- L_3 is the set of all words where a occurs only at even positions.
- L_4 is the set of all words where a occurs only at odd positions.
- L_5 is the set of all words of odd length.
- L_6 is the set of all words with an even number of a's.

Remark: For this exercise we assume that the first letter of a nonempty word is at position 1, e.g., $a \in L_4$, $a \notin L_3$.

Your task is to construct an FA, i.e., DFA or NFA or NFA-ε, for

$$L := (L_1 \setminus L_2) \cup \overline{(L_3 \triangle L_4) \cap L_5 \cap L_6} \text{ where } \triangle \text{ denotes the symmetric difference.}$$

while sticking to the following rules:

- You have to start from FAs for $L_1, \ldots, L_6$.
- Any further automaton has to be constructed from already existing automata via an algorithm introduced in the lecture, e.g., Bisim, BinOp, Comp, NFAtoDFA, etc.
- You are free to transform the propositional formula underlying the definition of L as you see fit.

Try to find an order on the construction steps which yields an FA for L with as few as possible states.

Exercise 2.2

For $n \in \mathbb{N}_0$ let msbf(n) be the language of all words over $\{0,1\}$ which represent n w.r.t. to the *most significant bit first* representation where an arbitrary number of leading zeros is allowed. For example:

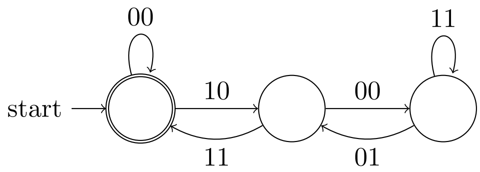
$$\text{msbf}(3) = \mathcal{L}(0^*11) \text{ and } \text{msbf}(0) = \mathcal{L}(0^*).$$

Similarly, let lsbf(n) denote the language of all *least significant bit first* representations of n with an arbitrary number of following zeros, e.g.:

$$\text{lsbf}(6) = \mathcal{L}(0110^*) \text{ and } \text{lsbf}(0) = \mathcal{L}(0^*).$$

(a) Construct and compare the minimal DFAs representing all even natural numbers $n \in \mathbb{N}_0$ w.r.t. the unary encoding (i.e., $n \mapsto a^n$), the msbf encoding, and the lsbf encoding.

(b) Consider the following FA $\mathcal{A}$ over the alphabet $\{00, 01, 10, 11\}$:



W.r.t. the msbf encoding, we may interpret any word $w \in \{00, 01, 10, 11\}^*$ as a pair of natural numbers $(X(w), Y(w)) \in \mathbb{N}_0 \times \mathbb{N}_0$. *Example*: (Underlined letters correspond to $Y(w)$.)

$$w = (00)^k 001011 \to (0\underline{0})^k 0\underline{0}1\underline{0}1\underline{1} \to (0^k011, 0^k001) \to (3,1) = (X(w), Y(w))$$

- Find constants $a, b \in \mathbb{Z}$ such that $aX(w) + bY(w) = 0$ for all $w \in \mathcal{L}(\mathcal{A})$.

(c) Construct the minimal DFA representing the language $\{w \in \{0,1\}^* \mid \text{msbf}^{-1}(w) \text{ is divisible by } 3\}$.