Technische Universität München **Winter term 2009/10**
I7
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Automata and Formal Languages – Homework 4

Due 19.11.2009.

Exercise 4.1

Show that for any natural number $n \geq 2$ there exist *minimal* DFAs $\mathcal{A}_1$ and $\mathcal{A}_2$, both having at most $n+1$ states, such that the *minimal* DFA accepting $\mathcal{L}(\mathcal{A}_1) \cup \mathcal{L}(\mathcal{A}_2)$ has at least n^2 states.

Exercise 4.2

Let $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ be an NFA.

We introduce a "game" on $\mathcal{A}$: We have two players, called "refuter" (short: $\bot$) and "prover" (short: $\top$). A play $\pi = \{s_0, t_0\}\{s_1, t_1\} \ldots$ of the two players on $\mathcal{A}$ is a sequence of (unordered) pairs of states. The initial pair $\{s_0, t_0\}$ can be chosen arbitrarily, but the choice of a successor pair $\{s_{i+1}, t_{i+1}\}$ is limited by the current pair $\{s_i, t_i\}$ as follows:

- If $s_i \in F \Leftrightarrow t_i \notin F$, then the play terminates immediately, and player $\bot$ is declared the winner of the play.
- If the pair $\{s_i, t_i\}$ has been visited before in the play, i.e., there is some $j < i$ s.t. $\{s_i, t_i\} = \{s_j, t_j\}$, then the play terminates and player $\top$ wins; otherwise:
- Two tokens are put on the states $\{s_i, t_i\}$. If $s_i = t_i$, then both tokens lie on the same state.

  Player $\bot$ then moves exactly one of the two tokens along an outgoing transition of the state the chosen token is located on.

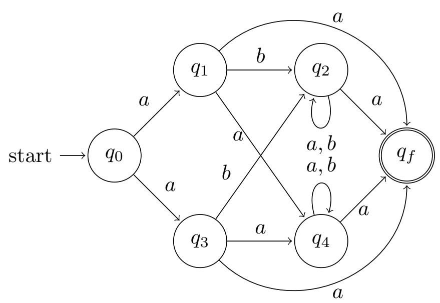
  If both states do not have outgoing transitions, i.e., if player $\bot$ cannot move, player $\top$ wins; otherwise:
- Let a be the label of the transition along which player $\bot$ has moved.

  Player $\top$ now has to try to match this move by moving the other token, i.e., the one that hasn't been moved in this round yet, along an outgoing transition also labeled by a.

  If player $\top$ cannot move in such a way, he immediately loses; otherwise:
- The new pair $\{s_{i+1}, t_{i+1}\}$ is determined by the states on which the two tokens are located.

A player wins a pair $\{s_0, t_0\}$ of states if he can choose his moves in such a way that he wins any resulting play starting in $\{s_0, t_0\}$.

(a) Consider the NFA of Ex3.4:



Determine which player wins the pair $\{q_1, q_2\}$, resp. the pair $\{q_4, q_f\}$, if there is a winner.

(b) Show for arbitrary NFAs that if $s_0 \sim t_0$, then player wins the pair $\{s_0, t_0\}$, otherwise player $\bot$ wins.

Hint: For the case that $s_0 \not\sim t_0$, consider the sequence of partitions $\sim_0, \sim_1, \ldots, \sim_l$ constructed by the algorithm "Bisim" (with $\sim = \sim_l$).