Technische Universität München **Winter term 2008/2009**
Theoretische Informatik **Exercise Sheet 12**
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Exercises "Automata and Formal Languages"

Exercise 12.1

Let $AP = \{p, q\}$ and let $\Sigma = 2^{AP}$. Give S1S and LTL formulas defining the following languages:

- $\{p, q\}\emptyset\Sigma^\omega$
  - We use free second-order variables A_1, A_2, A_3, and A_4 to denote the positions, where each of the four letters, $\emptyset$, $\{p\}$, $\{q\}$, and $\{p, q\}$ of Σ occurs. As in the construction of a formula from a Büchi automaton, we need the formula expressing that there is a unique letter at each position:

$$\psi = \forall x. \bigvee_{1\leq i\leq 4} (x \in A_i) \wedge \bigwedge_{1\leq i\leq 4} \Big(x \in A_i \Rightarrow \bigwedge_{j\neq i} x \notin A_j\Big)$$

    . Then we have as a solution:

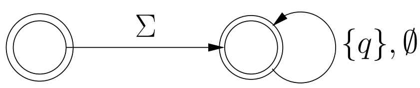
$$\psi \wedge 0 \in A_4 \wedge succ(0) \in A_0$$

  - $p \wedge q \wedge \mathsf{X}(\neg p \wedge \neg q)$
- $\Sigma^*\{q\}^\omega$
  - $\psi \wedge \exists x.\forall y.y \geq x \Rightarrow y \in A_3$
  - $\mathsf{FG}(q \wedge \neg p)$
- $\Sigma^*\{p\}\Sigma^*\{q\}\Sigma^\omega$
  - $\psi \wedge \exists x.\exists y.x < y \wedge x \in A_2 \wedge y \in A_3$
  - $\mathsf{F}(p \wedge \neg q \wedge \mathsf{XF}(q \wedge \neg p))$
- $\{p\}^*\{q\}^*\emptyset^*\Sigma^\omega$
  - ψ, because $\{p\}^*\{q\}^*\emptyset^*\Sigma^\omega = \Sigma^\omega$.
  - **true**

Exercise 12.2

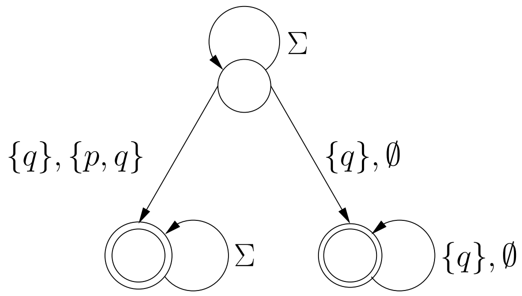
Let $AP = \{p, q\}$. Find Büchi automata accepting the languages defined by the following LTL formulas:

- $\mathsf{XG}\neg p$

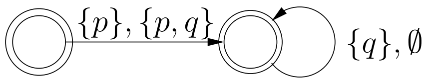


where $\Sigma = 2^{AP}$.

- $(\mathsf{GF}p) \Rightarrow (\mathsf{F}q)$. One should read the formula as *finitely many p's* or *eventually q*:



- $p \wedge \neg \mathsf{XF}p$

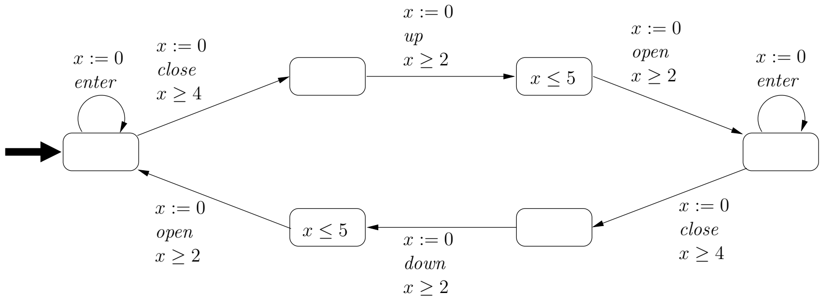


Exercise 12.3

Consider an autonomous elevator with the following behavior:

- The elevator operates between two floors, ground floor and first floor.
- Initially, the elevator is at the ground floor with its door open.
- Upon arrival at a certain floor, its door automatically opens. It takes at least 2 seconds from its arrival before the door opens but the door must definitely open within 5 seconds.
- Whenever the door is open, passengers can enter. They enter one by one, and we assume that the elevator has a sufficient capacity to accommodate any number of passengers.
- The door can close only 4 seconds after the last passenger entered.
- After the door closes, the elevator waits at least 2 seconds and then travels up or down to the other floor.

Design a timed automaton model of the elevator. Use the actions *up* and *down* to model the movement of the elevator, *open* and *close* to describe the door operation and the action *enter* which means that a passenger is entering the elevator. Provide two timed traces starting from the initial state.
Solution: Apart from the five actions, we use only one clock, x. Location invariants are shown within states, clock resets are written explicitly as an assignment, $x := 0$.



Two timed traces:

$$(1, \textit{enter})(5, \textit{close})(6, \textit{up})(11, \textit{open})(11, \textit{enter})\ldots$$
$$(0.2, \textit{enter})(0.3, \textit{enter})(0.3, \textit{enter})(100, \textit{close})(102.2, \textit{up})\ldots$$