Technische Universität München
Theoretische Informatik
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Exam "Automata and Formal Languages"

Exercise 1 **(3+4+3+4 points)**

Let $\Sigma = \{a, b\}$ be an alphabet and let $L_1, L_2 \subseteq \Sigma^*$ be languages. We define the *symmetric difference*, $L_1 \ominus L_2$, as follows:

$$L_1 \ominus L_2 = \{w \in \Sigma^* \mid w \text{ is in } L_1 \text{ or in } L_2 \text{ but not in both}\}$$

(a) Find two DFAs accepting $\mathcal{L}(a^*b)$ and $\mathcal{L}(ab^*)$, respectively. Make sure your DFAs are complete.

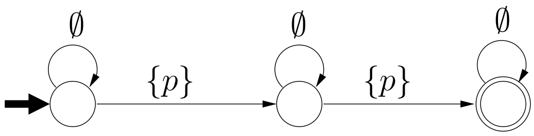
(b) Construct a DFA accepting $\mathcal{L}(a^*b) \ominus \mathcal{L}(ab^*)$.

(c) Find a regular expression r such that $\mathcal{L}(r) = \mathcal{L}(a^*b) \ominus \mathcal{L}(ab^*)$.

(d) Find an MSO(Σ) formula φ such that $L(\varphi) = \mathcal{L}(a^*b) \ominus \mathcal{L}(ab^*)$. *Hint:* You can solve (d) without having solved (a), (b), and (c).

Exercise 2 **(4 points)**

Let $\Sigma = 2^{\{p\}}$ and consider the following Büchi automaton $(\mathcal{A}, \mathcal{G})$.



Give an LTL formula ψ such that $L_\psi = \mathcal{L}(\mathcal{A}, \mathcal{G})$.

Exercise 3 **(3+3+3 points)**

Let Σ be an alphabet. For $\alpha \in \Sigma^\omega$ and $\sigma \in \Sigma$ we write $\mathsf{fin}(\sigma, \alpha)$ if σ occurs finitely often in α. Let $L(\Sigma) \subseteq \Sigma^\omega$ be the language $L(\Sigma) = \{\alpha \in \Sigma^\omega \mid \bigvee_{\sigma \in \Sigma} \mathsf{fin}(\sigma, \alpha)\}$

(a) Give a deterministic Muller automaton accepting $L(\{a, b\})$.

(b) Give an S1S formula $\psi(X_a, X_b, X_c)$ such that $L_\psi = L(\{a, b, c\})_{\{0,1\}}$.

(c) Give a deterministic Muller automaton accepting $L(\{a, b, c\})$ and argue its correctness.

Exercise 4 **(5 points)**

A *strict* Büchi automaton is syntactically like a Büchi automaton $(\mathcal{A}, \mathcal{G})$. It accepts a word $\alpha \in \Sigma^\omega$ if there exists a run ρ of $\mathcal{A}$ on α such that $\mathsf{inf}(\rho) = \mathcal{G}$. Show that the language $\{a^\omega, b^\omega\} \subset \{a, b\}^\omega$ is not accepted by any strict Büchi automaton.

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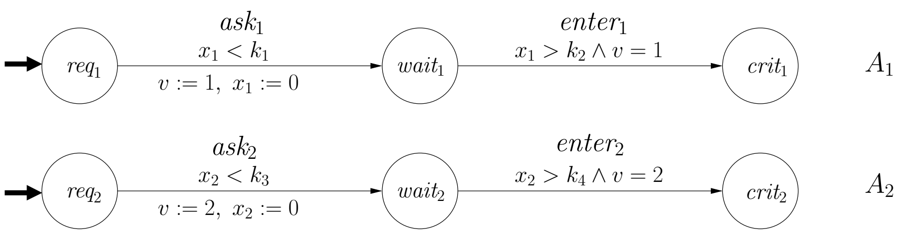
Exercise 5 (4+4 points)

(a) Consider the timed automata A_1, A_2, which operate on the set $\{x_1, x_2\}$ of clocks and the shared integer variable v. A_i has actions $\{ask_i, enter_i\}$ and locations $\{req_i, wait_i, crit_i\}$ for $i = 1, 2$. Guards are given on top of edges, reset and variable updates below edges. Initially, v has value 1. Moreover, $k_i \in \mathbb{R}^+$. Determine one value for each k_i for $i \in \{1, 2, 3, 4\}$ such that a state of the form $(crit_1, crit_2, v =?, x_1 =?, x_2 =?)$ is reachable in $A_1||A_2$. Give a witnessing sequence of transitions.



(b) Now consider the timed automata A_3 and A_4 below. They are similar to A_1 and A_2 with $k_i = 2$, but have $>$ replaced with $\geq$ in the guard on $enter_1$ and $<$ replaced with $\leq$ in the guard on ask_2. Give a timed trace of $A_3||A_4$ such that both action $enter_1$ and action $enter_2$ occur.