Technische Universität München
Theoretische Informatik
Prof. J. Esparza / Dr.-Ing. J. Kreiker

Winter term 2008/2009
Exercise Sheet 13
January 27, 2009

Test Exam "Automata and Formal Languages"

- This problem set serves as a preparation for the upcoming exam. It will not be corrected, but solutions will be discussed on Monday, February 2.
- Each problem is supposed to be solvable within 15-20 minutes time.
- You may use all your notes and other written material to solve the problems, since the exam will be *open book*.
- For MSO, S1S, and LTL, you may use all abbreviations introduced in class.
- Generally, final states are drawn with double circles, while initial states are marked by an incoming arrow.
- This problem set together with problem set 12, gives a good overview of possible exam exercises. It is not an exhaustive list of all possible kinds of exercises.

Exercise 1 **(4+2 points)**

Let $L_1, L_2 \subseteq \Sigma^*$ be languages.

(a) Suppose L_1 and L_2 are accepted by DFAs $A_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$ and $A_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$, respectively. Construct a DFA accepting $L_1 \setminus L_2$.

(b) Prove: If L_1 is regular, L_2 is not regular, and $L_1 \cap L_2 = \emptyset$, then $L_1 \cup L_2$ is not regular.

Exercise 2 **(3+3 points)**

Let $\Sigma = \{0, 1\}$. For $w \in \Sigma^*$, where $w = a_0a_1a_2 \ldots a_k$, we define $bin(w) := \Sigma_{i=0}^{k} a_i 2^i$. We define $R \subseteq \Sigma^* \times \Sigma^*$ as follows:

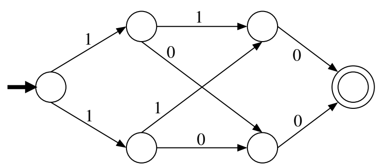
$$R = \{(w, w') \mid bin(w) > bin(w'),\ |w| = |w'|\}$$

(a) Show that R is a regular relation by finding an accepting transducer.

(b) Let $L = (\{0, 1\}\{0, 1\})^*$ be a language. Construct an automaton accepting $post_R(L)$.

Exercise 3 **(4+4 points)**

Consider the following NFA, A, over $\Sigma = \{0, 1\}$.

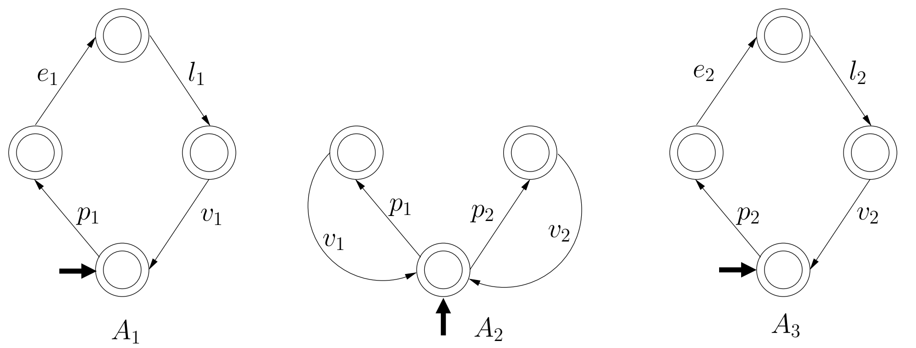


(a) Minimize A with respect to bisimilarity. Justify your solution.

(b) Construct a minimal DFA accepting $\mathcal{L}(A) \cap \{010, 011, 110, 111\}$.

Exercise 4 **(3+3 points)**

Consider the following three automata, A_1, A_2, and A_3 over the alphabets $\Sigma_1 = \{p_1, v_1, e_1, l_1\}$, $\Sigma_2 = \{p_1, v_1, p_2, v_2\}$, and $\Sigma_3 = \{p_2, v_2, e_2, l_2\}$, respectively.



(a) Construct the automaton B with $B = A_1||A_2||A_3$.

(b) Find an MSO($\Sigma_1 \cup \Sigma_2 \cup \Sigma_3$) formula, φ, such that $L(\varphi) = \mathcal{L}(B)$.

Exercise 5 **(3+3 points)**

Consider the following 2DFA, A, over $\{a, b\}$ with states $\{s_0, s_1, s_2\}$, all of which are final, and with initial state s_0. The transition table is given as follows:

	a	b
s_0	(s_0, R)	(s_1, R)
s_1	(s_1, R)	(s_2, L)
s_2	(s_0, R)	(s_2, L)

(a) Give a regular expression, r, such that $\mathcal{L}(r) = \mathcal{L}(A)$.

(b) Give a formula, ψ of MSO($\{a, b\}$) such that $L(\psi) = \mathcal{L}(A)$.

Exercise 6 **(3+3+2 points)**

Let $\Sigma = \{a, b\}$ and let $L_1, L_2 \subseteq \Sigma^\omega$ be an alphabet and two languages, such that

- $L_1 = \{\alpha \mid a \text{ occurs infinitely often in } \alpha\}$,
- $L_2 = \{\alpha \mid b \text{ occurs infinitely often in } \alpha\}$, and
- $L = L_1 \cap L_2$

(a) Find a Büchi automaton accepting L.

(b) Find a Muller automaton accepting L.

(c) Find an S1S formula, $\psi(A_a, A_b)$, such that $L_\psi = L_{\{0,1\}}$.

Exercise 7 **(2+4 points)**

Let Σ be an alphabet Let $\mathcal{A} = (S, \rightarrow, S_{in})$ be an automaton over Σ and let $\mathcal{G} \subseteq S$ be a set of states. A universal Co-Büchi automaton $(\mathcal{A}, \mathcal{G})$ accepts an infinite word, $\alpha \in \Sigma^\omega$, if for *all* runs ρ of $\mathcal{A}$ on α holds that $\mathsf{inf}(\rho) \cap F = \emptyset$. The set of all words accepted by a universal Co-Büchi automaton $(\mathcal{A}, \mathcal{G})$ is written $L^{uc}(\mathcal{A}, \mathcal{G})$.

(a) Let $\Sigma = \{a, b\}$ and assume

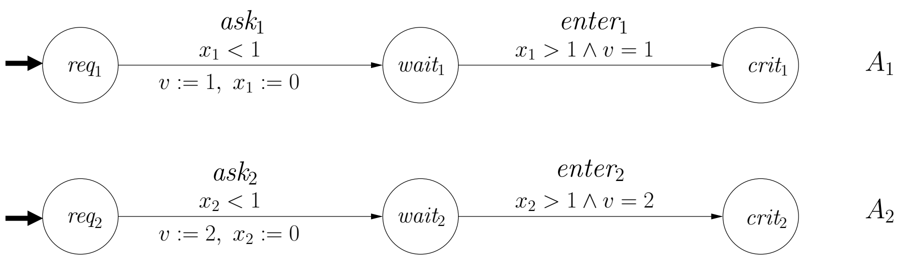
$$\mathcal{A}' = (\{q_0, q_1\}, \{(q_0, a, q_0), (q_0, b, q_0), (q_0, b, q_1), (q_1, b, q_1)\}, \{q_0\})$$
$$\mathcal{G}' = \{q_1\}$$

Give an ω-regular expression describing $\mathcal{L}^{uc}(\mathcal{A}', \mathcal{G}')$.

(b) Prove: $L \subseteq \Sigma^\omega$ is ω-regular if and only if it is accepted by a universal Co-Büchi automaton.

Exercise 8 **(4+2+2+2 points)**

Consider the timed automata A_1 and A_2 given below. They operate on the set $\{x_1, x_2\}$ of clocks and the shared integer variable v. A_i has actions $\{ask_i, enter_i\}$ and locations $\{req_i, wait_i, crit_i\}$ for $i = 1, 2$. Guards are given on top of edges, reset and variable updates below edges. Initially, v has value 1.



(a) Construct $A_1||A_2$.

(b) Find a timed trace of the system $A_1||A_2$ that ends with an action $enter_2$ and no more action is possible after that.

(c) Give a sequence of transitions in $A_1||A_2$ starting from the inital state, such that the actions ask_1, ask_2, and $enter_2$ occur in that order.

(d) Argue that no state of the form $(crit_1, crit_2, \ldots)$ is reachable in $A_1||A_2$.