Technische Universität München
Theoretische Informatik
Prof. J. Esparza / Dr.-Ing. J. Kreiker

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Exercise Sheet 11
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Exercises "Automata and Formal Languages"

Exercise 11.1

Find the ω-regular languages L_{ψ_1} and L_{ψ_2} associated with the following S1S formulas:

- $\psi_1(X, Y, Z, s) = \forall x.(s = succ(0) \wedge (x \in X \Leftrightarrow x \in Z)) \vee (s = 0 \wedge (x \in Y \Leftrightarrow x \in Z))$
- $\psi_2(x, y) = \exists X.succ(0) \in X \wedge x \in X \wedge y \in X \wedge \forall z.(z \in X \Leftrightarrow succ(succ(z)) \in X)$

Exercise 11.2

Let $\Sigma = \{a, b, c\}$ and consider $L, L' \subseteq \Sigma^\omega$ as defined below. Find S1S formulas φ and φ' defining $L_{\{0,1\}}$ and $L'_{\{0,1\}}$, respectively.

- L is the language, where there is no b after an a.
- L' is the language, where no c occurs at an odd position.

Exercise 11.3

Let $k_1, k_2 \in \mathbb{N}_0$ be constants. Find a Presburger arithmetic formula, $\varphi(x, y)$, with free variables x and y such that $[x \mapsto n_x, y \mapsto n_y] \models \varphi(x, y)$ iff $n_x \geq n_y$ and $(n_x - n_y) \equiv k_1 \pmod{k_2}$. Find a corresponding Büchi automaton for the case $k_1 = 0$ and $k_2 = 2$. Note that $[x \mapsto n_x, y \mapsto n_y]$ is an interpretation assigning n_x to x and n_y to y.