

## Exercises “Automata and Formal Languages”

### Exercise 9.1

Show that  $\{a, b\}^\omega$  is not countable. *Hint:* A set,  $M$ , is countable, iff there exists a surjective function  $f : \mathbb{N} \rightarrow M$ .

### Exercise 9.2

For each  $L_1, L_2 \subseteq \{a, b\}^\omega$  below, find an  $\omega$ -regular expression of the form  $\bigcup_{i=1}^n U_i V_i^\omega$  representing the language, such that each  $U_i$  and  $V_i$  are regular languages of finite words.

- $L_1 = \{\alpha \mid k \text{ is even for each substring } ba^k b \text{ of } \alpha\}$
- $L_2 = \{\alpha \mid \alpha \text{ has no occurrence of } bab\}$

### Exercise 9.3

Use the product construction to obtain a Büchi automaton accepting  $L_1 \cap L_2 \cap L_3 \subseteq \{a, b\}^\omega$ , where

- $L_1 = \{\alpha \mid \text{infinitely many } a\text{'s occur in } \alpha\}$
- $L_2 = \{\alpha \mid \text{finitely many } b\text{'s occur in } \alpha\}$
- $L_3 = \{\alpha \mid \text{each } a \text{ in } \alpha \text{ is immediately followed by a } b\}$

### Exercise 9.4

Show that *deterministic* Büchi automata are closed under union and intersection.