Technische Universität München **Winter term 2008/2009**
Theoretische Informatik **Exercise Sheet 7**
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Exercises "Automata and Formal Languages"

Exercise 7.1

Define MSO formulas (i) $Sing(X)$, (ii) $X \subseteq Y$, (iii) $X \subseteq Q_a$, and (iv) $succ(X, Y)$ meaning that (i) X is a singleton set, (ii) X is a subset of Y, (iii) all $x \in X$ satisfy Q_a, and (iv) X and Y are singletons, $\{x\}$ and $\{y\}$, such that y is the successor of x. Translate the formula

$$\exists Z \forall x (Q_a(x) \rightarrow \exists y (succ(x, y) \wedge y \in Z))$$

into an equivalent one using (i)-(iv) but *no* first-order variables.

Exercise 7.2

What languages are defined by the following MSO sentences?

- $\exists x\ zero(x)$
- $\forall x\ zero(x)$
- $\forall X \forall x\ x \in X \vee x \notin X$

Exercise 7.3

Characterize the language, L, described by the following formula ψ. Also give an automaton, A, such that $\mathcal{L}(A) = L$.

$$\begin{aligned}\psi \quad = \quad & \big(\neg \exists x \exists y (succ(x, y) \wedge Q_a(x) \wedge Q_b(y))\big) \ \wedge\ \big(\forall x (Q_b(x) \rightarrow \exists y (succ(x, y) \wedge Q_a(y)))\big) \\ & \wedge\ \big(\exists x (\neg \exists y succ(x, y) \wedge Q_a(x))\big)\end{aligned}$$

Exercise 7.4

Come up with MSO sentences defining the following languages over $\{a, b\}$.

- $\mathcal{L}((aa)^*)$. Use the construction from the proof.
- The set of words, where between each two b's with no other b in between there is a block of an odd number of letters a.
- The set of words, where a occurs only at even, or only at odd positions.
- The set of words with odd length and an odd number of occurrences of a.

Exercise 7.5

Recall star-free languages from Exercise 1.3 and fix an alphabet $\Sigma = \{a, b, c\}$.

- Give an MSO sentence *without set variables* defining the star-free language $\Sigma^* ab \overline{\Sigma^* a \Sigma^*}$.
- Show that for *every* star-free language, there exists an MSO sentence *without set variables* defining it.