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Theoretische Informatik
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Exercise Sheet 6
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Exercises "Automata and Formal Languages"

Exercise 6.1

Implement the crossing sequence construction and apply your implementation to the 2DFA of Exercise 5.3.

Exercise 6.2

Define the n-ary asynchronous product of n NFAs, such that product states are n-element sets rather than n-tuples! Show that the thus defined n-ary product is associative and commutative.

Exercise 6.3

In this exercise you are supposed to design a k-bit binary counter, C, of k communicating automata $A_0, \ldots, A_{k-1}$, such that $C = A_0 || \ldots || A_{k-1}$. Each automaton represents one bit. The alphabet of automaton A_i must contain (at least) the letters '$0[i]$' and '$1[i]$'. If the letter $a_i[i]$ is read, it means that the i-th bit currently has value a_i. Whenever a letter 'inc' is read, the number represented by the automata must be incremented by 1 (by a number of communications among the k automata). Design the automata A_i such that C satisfies the following specification:

- $\mathcal{L}(C) |_{\{inc\}} = inc^*$
- For all $j \geq 0$: $inc^j a_0[0] a_1[1] \ldots a_{k-1}[k-1] \in \mathcal{L}(C) |_{\{inc, 0[i], 1[i] \mid 0 \leq i < k\}}$ if and only if $j \mod 2^k = \sum_{i=0}^{k-1} a_i[i] 2^i$

Exercise 6.4

In this exercise you are supposed to design a railway controller. Consider a circular railway of 8 segments: $0 \to 1 \to \ldots \to 7 \to 0$. There are three trains modeled by the automata T_0, T_1, and T_2. Each automaton T_i has the states $Q_i = \{q_{i,0}, \ldots, q_{i,7}\}$, the alphabet $\Sigma_i = \{enter[i,j] \mid 0 \leq j \leq 7\}$, the transition relation $\delta_i = \{(q_{i,j}, enter[i, j +_8 1], q_{i,j+_8 1}) \mid 0 \leq j \leq 7\}$, and the initial state $q_{i,2i}$, where $+_8$ is addition modulo 8.

Define a family $C_1, \ldots C_k$ of controllers to make sure that two trains can never be on adjacent segments, while all trains can eventually move, if they want to. Formally, the product $R = C_1 || \ldots || C_k || T_1 || T_2 || T_3$ shall satisfy the following specification:

- For $i = 0, 1, 2$: $\mathcal{L}(R) |_{\Sigma_i} = (enter[i, 2i] enter[i, 2i +_8 1] \ldots enter[i, 2i +_8 7])^*$
- For all $w enter[i,j] enter[i',j'] w' \in \mathcal{L}(R) |_{\{enter[i,j] \mid i=0,1,2; 0 \leq j \leq 7\}}$ holds that $i' \neq i$ implies $|j - j'| \notin \{0, 1, 7\}$

Hint: Typical values of k could be 1, 3, or 8 (where 8 is preferred).