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Theoretische Informatik
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Exercise Sheet 4
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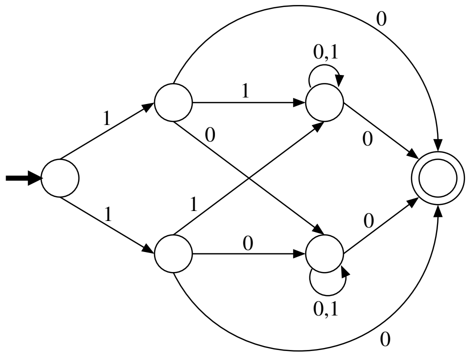
Exercises "Automata and Formal Languages"

Exercise 4.1

Apply the partitioning algorithm to the following NFA and construct the resulting NFA that is minimal with respect to bisimulation.



Exercise 4.2

Let $\Sigma = \{0, 1\}$ be our alphabet. We write $\#b$ to denote the interpretation of $b \in \Sigma^*$ as a binary number, possibly with leading zeros. Construct minimal DFAs accepting the bounded languages L_1, L_2, and L_3^2 defined below.

- $L_1 = \{b \mid \#b \mod 3 = 0\} \cap \Sigma^4$.
- $L_2 = \{b \mid \#b \text{ is prime}\} \cap \Sigma^4$.
- $L_3^k = \{ww \mid w \in \Sigma^k\}$. How many states has a minimal DFA accepting L_3^k?.

Use the minimal intersection algorithm for bounded languages to construct a DFA accepting $L_2 \cap L_3^2$.

Exercise 4.3

(a) For any $k > 0$ and $L \subseteq \Sigma^k$ we define $\widehat{L}^k$ to be the language $\Sigma^k \setminus L$. Given a minimal DFA accepting L, construct a minimal DFA accepting $\widehat{L}^k$.

(b) Given two minimal DFAs accepting bounded languages L_1 and L_2 with words of length k, construct a minimal DFA accepting $L_1 \cup L_2$.

(c) For any language $L \subseteq \{0, 1\}^k$ of binary numbers of length k, we define $L + 1$ to be the language $\{w + 1 \mod 2^k \mid w \in L\}$. Construct a minimal DFA accepting $L + 1$ from a minimal DFA accepting L.

(d) Let $A = (Q, \{0, 1\}, \delta, q_0, F)$ be a minimal DFA such that $\mathcal{L}(A)$ is a bounded language of binary numbers. What language is accepted by the automaton $A' = (Q, \{0, 1\}, \delta', q_0, F)$, where $\delta'(q, b) = \delta(q, 1 - b)$?