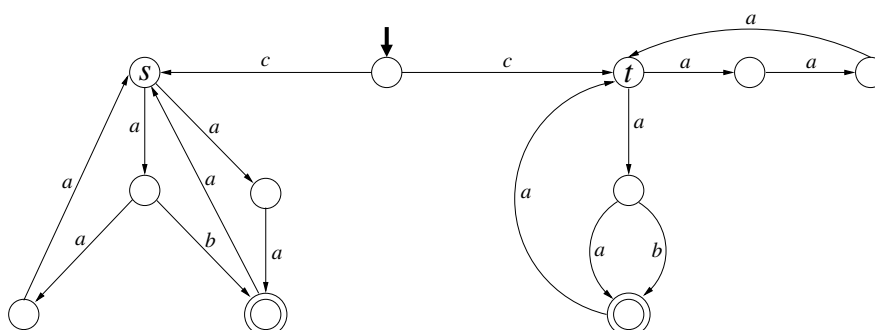


## Exercises “Automata and Formal Languages”

### Exercise 3.1

Consider the following NFA and show that  $s \sim t$ . Compute the relations  $\sim_0, \sim_1, \sim_2$ , and  $\sim_3$  and construct the minimal NFA with respect to bisimulation.



### Exercise 3.2

Let  $R$  be a bisimulation relation. Must  $R$  be reflexive? Symmetric? Transitive?

### Exercise 3.3

Let  $A = (Q, \Sigma, \delta, q_o, F)$  be an NFA and let  $A_q = (Q, \Sigma, \delta, q, F)$  for any  $q \in Q$ . Prove or disprove: If  $\mathcal{L}(A_q) = \mathcal{L}(A_{q'})$  then  $q \sim q'$ .

### Exercise 3.4

Let  $A = (Q, \Sigma, \delta, q_o, F)$  be an NFA. A relation  $R \subseteq Q \times Q$  is called a *simulation* iff  $q_1 R q_2$  implies

- $q_1 \in F \iff q_2 \in F$  and
- for every  $a \in \Sigma$  and every  $q'_1 \in \delta(q_1, a)$  there exists a  $q'_2 \in \delta(q_2, a)$  such that  $q'_1 R q'_2$ .

The relation  $\preceq = \bigcup \{R \mid R \text{ is a simulation}\}$  is the largest simulation. If  $q \preceq q'$  we say that  $q$  is simulated by  $q'$ .

- Show that  $q \preceq q'$  implies  $\mathcal{L}(A_q) \subseteq \mathcal{L}(A_{q'})$ .
- Does  $q \preceq q'$  and  $q' \preceq q$  imply  $q \sim q'$ ?